Cox rings of smooth projective surfaces

Michela Artebani

The Cox ring of a smooth projective surface X with free and finitely generated Picard group is the $\mathrm{Pic}(X)$-graded ring which can be defined as

$$\mathcal{R}(X) := \bigoplus_{[D]\in \mathrm{Pic}(X)} H^0(X, \mathcal{O}_X(D)).$$

It is an open problem to classify which surfaces have finitely generated Cox ring and a presentation for the ring has been computed explicitely only in some special cases. In this talk we will give a general picture of the classification problem and we will present a recent result, joint with A. Garbagnati and A. Laface, about Cox rings of rational elliptic surfaces.

Dynamical degrees of birational transformations of surfaces

Serge Cantat

Given a birational transformation f of a surface X, its dynamical degree is the limit $L(f)$ of the sequence $(\deg(f^n(C))^{1/n})$, where C is any ample divisor of X. I shall describe new results on the possible values of $L(f)$, when f runs over the set of all birational transformations of surfaces. This set of positive numbers is a well ordered set of algebraic numbers; all of them are either equal to 1, to a Salem number, or to a Pisot number.

TBA

François Charles

K3 surfaces with a non-symplectic automorphism and product-quotient surfaces by certain cyclic groups.

Alice Garbagnati

The goal of this talk is the classification of K3 surfaces which are minimal models of the quotient of the product of two curves, $C_1 \times C_2$, by the automorphism $g_1 \times g_2$ where g_i is an automorphism of the curve C_i of order either p or $2p$ (p is a prime). Each K3 surface of this type clearly admits a non-symplectic automorphism of order p, induced either by $1 \times g_2$ or by $1 \times g_2^2$ respectively. The K3 surfaces admitting a non-symplectic automorphism of order p are classified and their parametrizing space consists of a finite number of connected components, which dipend both on the order of the automorphism and on the fixed locus of the automorphism on the surface. Thus, we compare this classification with the classification of the K3 surfaces minimal models of $C_1 \times C_2/g_1 \times g_2$ and it turns out that a big part of the K3 surfaces with a non-symplectic automorphism are in fact minimal models of $C_1 \times C_2/g_1 \times g_2$. This gives a geometrical interpretation of the peculiarity of K3 surfaces with a non-symplectic automorphism of order p and of the fact they often admits a non-symplectic automorphism of order

2p too. The results presented are obtained in collaboration with Matteo Penegini.

Cycles and curves on K3 surfaces

Daniel Huybrechts

I will discuss examples and some properties of curves on K3 surfaces on which all points are rationally equivalent (as points on the surface).

k-gonal loci in Severei varieties of curves on $K3$ surfaces

Andreas Knutsen

This is a report on joint work with C. Ciliberto. Let (S, H) be a general primitively polarized complex $K3$ surface (S, H) of genus p and consider the Severi varieties $V_{|H|,\delta}$ of δ-nodal curves in $|H|$, for $0 \leq \delta \leq p$. It is well-known that $V_{|H|,\delta}$ is smooth and nonempty of dimension $p - \delta$. We consider the subloci $V^k_{|H|,\delta}$ of curves whose normalizations possess a g^1_k. We give necessary and sufficient conditions depending on p, δ and k for these loci to be nonempty, and prove that, when nonempty, there is always an irreducible component of the expected dimension $\min\{2(k-1), p-\delta\}$. In contrast to the case of smooth curves, the Severi varieties contain proper subloci of gonalities lower than the maximal gonality given by Brill-Noether theory.

I will also discuss results that are valid on any complete linear system $|H|$ and relations with and applications to rational curves in the punctual Hilbert scheme $Hilb^k(S)$.

The rationality of the moduli spaces of Coble surfaces and of nodal Enriques surfaces.

Shigeyuki Kondo

We prove the rationality of the coarse moduli spaces of Coble surfaces and of nodal Enriques surfaces over the field of complex numbers. This is a joint work with Igor Dolgachev.

Multiplicative excellent families of elliptic surfaces of type E_7 or E_8

Abhinav Kumar

Oguiso and Shioda described the possible Mordell-Weil lattices that could arise for rational elliptic surfaces over $\overline{\mathbb{Q}}$. The study of specific Mordell-Weil lattices has connections to invariant theory and inverse Galois theory (for instance, of some Weyl groups of root lattices), and Shioda has used these techniques to construct "excellent" families of rational elliptic surfaces with additive reduction and Mordell-Weil lattice E_8, E_7, E_6, D_4 etc, and also for multiplicative reduction and Mordell-Weil lattice E_6.

We describe joint work with Shioda which deals with the multiplicative reduction case with Mordell-Weil lattice E_8 or E_7. The parameters of the "excellent" families are related to the fundamental multiplicative invariants of the corresponding Weyl groups. We use our results to produce examples of elliptic surfaces for which the splitting field has large Galois group, and also examples for which it has trivial Galois group (all sections defined over $\mathbb{Q}$).

Geometry and Arithmetic of degenerations of K3 surfaces

Radu Laza

An important open problem is the construction of a geometric compactification for the moduli of polarized K3 surfaces. Since the moduli of polarized K3 surfaces is a locally symmetric variety (the quotient of a Type IV domain by an arithmetic group), it is natural to expect that a geometric compactification will be of toroidal or semi-toric type; this is not yet known. In this talk, I will employ a different point of view, based on the minimal model program, to shed some light on the subtle interplay between the geometry and arithmetic of degenerations of K3 surfaces.

Arithmetic Moduli of Enriques Surfaces

Christian Liedtke

We construct the moduli space of Enriques surfaces over the integers. On our way, we prove lifting of Enriques surfaces to characteristic zero, and describe geometric models of the K3(-like) double cover of an Enriques surface. The whole description is characteristic-free and gives a natural and complete picture, even in characteristic 2.

The rationality of moduli spaces of Eisenstein K3 surfaces

Hisanori Ohashi

An Eisenstein K3 surface is a K3 surface equipped with a non-symplectic automorphism of order 3. The properties of such K3s were studied recently by Artebani, Sarti, Taki. In this talk we will discuss the techniques for the proof of the rationality of the moduli spaces of these Eisenstein K3 surfaces; since the moduli spaces are arithmetic quotients of Hermitian domains, we need to connect them with algebraic moduli spaces via period maps and the global Torelli theorem. This is a joint work with S. Ma, and S. Taki.

Paracanonical systems of varieties of maximal Albanese dimension

Rita Pardini

I will report on some recent joint work with M. Mendes Lopes and G.P. Pirola.

Let X be a smooth complex projective variety of irregularity $q > 0$, and let H be an irreducible family of effective divisors of X that dominates a component of the group $Pic(X)$: given a divisor D algebraically equivalent to the elements of H, we give a cohomological criterion to ensure that D belong to H. By applying this criterion to the study of the main paracanonical system of a variety of general type with generically finite Albanese map, we are able to refine results due to Beauville in the case of surfaces and to Lazarsfeld and Popa in higher dimension. In particular, if the dimension of X is > 2 we obtain an unexpected inequality between the numerical invariants of X, under the assumption that X has generically finite Albanese map and does not have fibrations of a certain type.

Petri map and Brill-Noether theory for algebraic surfaces.

Gianpietro Pirola

Let X be a projective variety of maximal Albanese dimension, let K be the canonical line bundle and let D be an effective divisor of X. The Petri map (defined in analogy with the case of curves) is the multiplication map $m : H^0(D) \times H^0(K-D) \longrightarrow H^0(K)$. We discuss the relations of m with Brill-Noether theory, with generic vanishing theorems and with the map (studied by Xiao for $\dim X = 2$) that sends a wedge product of global 1-forms on X to its restriction to D. We obtain lower bounds for $h^0(K_D)$ when D moves linearly on X. In this way we sharpen and generalize a result of Xiao. We also discuss and prove an existence result of Brill-Noether type in the case in which X is a surface. The results are a joint work with M. Mendes Lopes and R. Pardini

Pluricanonical maps of stable surfaces

Sonke Rollenske

Stable surfaces occur naturally in the compactification of the Giesecker moduli spaces of surfaces of general type. We extend classical results of Kodaira and Bombieri to this setting: if X is a stable surface and I is its global index, then $4IK_X$ is base-point-free and $8IK_X$ is very ample.

If time permits I will also present some examples and discuss the geography problem for Gorenstein stable surfaces. This is joint work with Wenfei Liu.

Supersingular K3 surfaces with Artin invariant 10 (joint work with S. Kondo)

Ichiro Shimada

By means of a certain duality between the Picard lattices of supersingular K3 surfaces of Artin invariants sigma and 11-sigma, we classify genus one fibrations on supesingular K3 surfaces with Artin invariant 10 in characteristic 2 and 3, and investigate the automorphism group of these supersingular K3 surfaces.

The surface of cuboids and Siegel modular threefolds

Damiano Testa

A perfect cuboid is a parallelepiped with rectangular faces all of whose edges, face diagonals and long diagonal have integer length. A question going back to Euler asks for the existence of a perfect cuboid. No perfect cuboid has been found, nor it is known that they do not exist.

In this talk I will talk about Siegel modular threefolds: these are certain moduli spaces of abelian surfaces with level structure. Then, I will proceed to show that the space of cuboids is a divisor in one of these moduli spaces. Therefore the existence of a perfect cuboid is equivalent to the existence of special torsion structures in abelian surfaces defined over number fields.

Explicit transcendental Brauer classes on K3 surfaces and arithmetic applications

Tony Varilly-Alvarado

Transcendental elements of the Brauer group of an algebraic variety, i.e., Brauer classes that remain nontrivial after extending the ground field to an algebraic closure, are quite mysterious from an arithmetic point of view. These classes do not arise for curves or surfaces of negative Kodaira dimension. In 1996, Harari constructed a 3-fold with a transcendental Brauer-Manin obstruction to the Hasse principle. Until recently, his example was the only one of its kind. We show that transcendental elements of the Brauer group of an algebraic surface can obstruct the Hasse principle and weak approximation. For example, we construct a general K3 surface X of degree 2 over $\mathbb{Q}$, together with a two-torsion Brauer class alpha that is unramified at every finite prime, but ramifies at real points of X. Motivated by Hodge theory and work of van Geemen, the pair (X, α) is constructed from a double cover of $\mathbb{P}^2 \times \mathbb{P}^2$ ramified over a hypersurface of bi-degree $(2, 2)$. This is joint work with Brendan Hassett.