## WORKSHOP ALGEBRAIC SURFACES AND RELATED TOPICS POITIERS 6–7 OCTOBER 2011

#### Non-uniqueness of Fourier-Mukai kernels Alberto Canonaco (Pavia)

We will discuss some properties of the functor sending an object in the bounded derived category of the product of two smooth projective varieties to the Fourier-Mukai functor with that object as kernel. In particular we will show that this functor is not always essentially injective, namely that there exist non-isomorphic kernels defining isomorphic Fourier-Mukai functors. On the other hand, the cohomology sheaves of a kernel are always uniquely determined, up to isomorphism, by the functor. This is a joint work with P. Stellari.

## A new invariant of Fano manifolds Cinzia Casagrande (Torino)

We will talk about some properties of the Picard number  $\rho_X$  of a Fano manifold X. More precisely, we will introduce an integral invariant of X which "measures" the difference of Picard numbers  $\rho_X - \rho_D$ , where D is any prime divisor in X, and explain some results about this invariant. In particular, we always have  $\rho_X < \rho_D + 9$ , and if  $\rho_X > \rho_D + 3$ , then X is the product of a surface with another manifold.

# Prym map and second Prym canonical Gaussian map Elisabetta Colombo (Milano)

We show that the second fundamental form of the Prym map is a lifting of the second Gaussian map of the Prym canonical bundle. As a corollary we obtain some curvature properties of the Prym locus. This generalizes analogous results about the second fundamental form of the period map for curves and the second Gaussian map for the canonical bundle. This is a joint work with Paola Frediani

## Severi varieties of K3 surfaces Thomas Dedieu (Toulouse)

Let X be a surface. The Severi varieties of X parametrize reduced irreducible nodal curves of a given genus in a specified linear system |L| on X. I shall explain that in case X is K3 and the genus is positive, every reduced irreducible curve of |L| lies in the Zariski closure of the relevant Severi variety. Next I will prove the irreducibility of universal Severi varieties parametrizing hyperplane sections of primitve K3 surfaces of genus  $g, 3 \le g \le 11, g \ne 10$ .

# Existence of rational curves on Calabi-Yau threefolds with special divisors Simone Diverio (Paris Jussieu)

It is known since the works of Wilson in the late 80s that the existence of certain special divisors on a Calabi-Yau threefold implies the existence of rational curves on it. For instance, Peternell and later Oguiso showed that the existence of a non-zero effective non-ample divisor suffices. We shall show that a similar result holds true if one merely supposes the existence of a non-zero nef non-ample divisor, provided the second Betti number of the Calabi-Yau threefold is greater than 4. This is a joint work with A. Ferretti.

# Rank properties of the second canonical and Prym-canonical Gaussian maps Paola Frediani (Pavia)

We will discuss rank properties of the second Gaussian map of Prym-canonical line bundles on a curve C, i.e. line bundles of the form  $K_C \otimes A$ , where A is an element of order two in  $Pic^0(C)$ . In particular we will show that these maps are surjective for the general point in the moduli space  $\mathcal{R}_g$  parametrizing isomorphism classes of pairs (C, A) where the genus of the curve C is at least 20 and A is a two-torsion line bundle. This generalizes an analogous result of Calabri, Ciliberto and Miranda on the second Gaussian map of the canonical line bundle for the general curve of genus at least 18. We will also describe some degeneracy loci for both Prym-canonical and canonical second Gaussian maps. This is a joint work with Elisabetta Colombo.

## Rational curves and bounds on the Picard number of Fano manifolds Carla Novelli (Padova)

A generalization of a conjecture of Mukai involving the Picard number  $\rho_X$ and the pseudoindex  $i_X$  of a Fano manifold X states that  $\rho_X(i_X - 1) \leq \dim X$ with equality iff  $X = (\mathbb{P}^{i_X-1})^{\rho_X}$ . We prove that this conjecture holds if X has pseudoindex  $i_X > \frac{\dim X}{3}$ , or if X has pseudoindex  $i_X = \frac{\dim X}{3}$  and admits an unsplit dominating family of rational curves.

## Properties of some moduli spaces of semistable sheaves on K3 and abelian surfaces Arvid Perego (Nancy)

Moduli spaces of sheaves on K3 and abelian surfaces are one of the main tools to produce examples of irreducible symplectic manifolds. In this talk we deal with some of them, in particular those singular moduli spaces admitting a symplectic resolution, and we describe some of their properties.

## Singular moduli spaces of sheaves on K3 surfaces Antonio Rapagnetta (Roma Tor Vergata)

Irreducible holomorphic symplectic manifolds play a central role in the classification of compact kaehler varieties with trivial canonical bundle. Up to deformation equivalence very few examples are known. About 25 years ago Mukai suggested to construct examples starting from moduli spaces of sheaves on projective surfaces with trivial canonical bundle. We will focus on the geometry of the irreducible holomorphic symplectic manifolds that can be obtained by desingularizing singular moduli spaces of sheaves on those surfaces.

# Fourier-Mukai functors in the supported setting Paolo Stellari (Milano)

Fourier-Mukai functors play a distinct role in algebraic geometry. Nevertheless two questions remained open: are all exact functors between the bounded derived categories of smooth projective varieties of Fourier-Mukai type? Is the Fourier-Mukai kernel unique? We will answer positively to these questions under some assumptions on the exact functor. This extends previous results by Lunts, Orlov and Ballard. Along the way, we will show that, in geometric contexts, full functors are faithful as well. This is a joint work in collaboration with A. Canonaco and, partly, with D. Orlov.