Galois Closure of a Degree 5 Covering and Decomposition of Its Jacobian Variety

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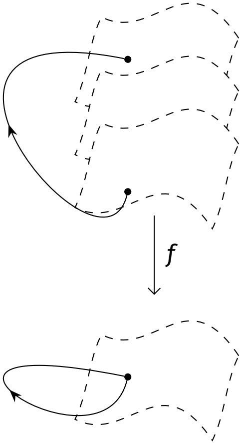
Outline

Section 1

Preliminaries

Galois Closure of a Covering Map



Action of $\pi_1(Y - B)$ on a fiber of f.

Notation

Set two compact Riemann surfaces X and Y and a degree d covering map $f: X \to Y$. Denote by $\rho: \pi_1(Y - B) \to S_d$ its monodromy representation, where B is the set of branch values of f; and set $G \coloneqq \operatorname{im} \rho$, its monodromy group.

Definition (Galois closure)

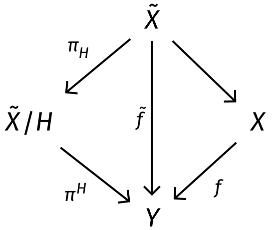
The Galois closure $\tilde{f}: \tilde{X} \to Y$ of f is the minimal regular covering of Y which factors trough f.

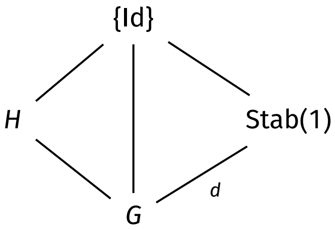
Remark

The group of automorphisms $\operatorname{Aut}(\tilde{f})$ is isomorphic to G.

Intermediate Coverings of the Galois Closure

Each intermediate covering of $\tilde{f}$ is induced by the action of a subgroup H of G. In particular, the map f corresponds to $\mathrm{Stab}(1)$ as in the following diagrams:





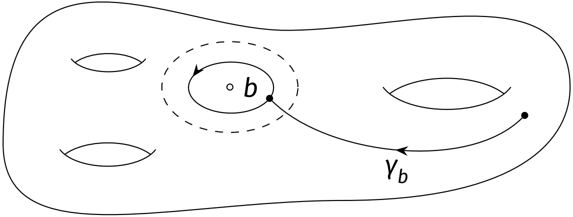
Moreover, there are natural correspondences:

$$\deg \pi_H = |H|, \qquad \deg \pi^H = [G : H] \qquad \text{and} \qquad \mathrm{Aut}\left(\pi^H\right) \cong \frac{\mathrm{N}_G(H)}{H}.$$

Small Loops

Notation

Set a branch value $b \in B$. A small loop around b is denoted by γ_b.



Small loop γ_b around b.

Lemma

If $\rho(\gamma_b)$ has cycle structure $[m_1, \dots, m_k]$, then b has exactly k preimages by f, with multiplicities $m_1, \dots, m_k$.

Definition

The type of b is $[m_1, \dots, m_k]$. Also, b is called even or odd according to the parity of $\rho(\gamma_b)$.

Prym Variety of a Covering

Notation

The Jacobian variety of a curve Z is denoted by $(\mathrm{J}(Z), \Theta_Z)$. The homomorphism $f^* \mathrm{J}(Y) \to \mathrm{J}(X)$ has finite kernel.
The complement of $f^* \mathrm{J}(Y)$ in $\mathrm{J}(X)$ is the *Prym variety* of f and is denoted by $\mathrm{P}(f)$. The polarizations induced by Θ_X on $f^* \mathrm{J}(Y)$ and $\mathrm{P}(f)$ are denoted by $\Theta_{f^* \mathrm{J}(Y)}$ and $\Theta_{\mathrm{P}(f)}$, respectively.

Theorem (Recillas and Rodríguez (2003))

- *The polarization* $(f^*)^* \Theta_X$ *is analytically equivalent to* $\Theta_Y^{\otimes d}$.
- *The group* $\ker f^*$ *is isotropic in* $\mathrm{J}(Y)[d]$ *with respect to the Weil form associated to* $\Theta_Y^{\otimes d}$.
- *The map* f^* *induces an isomorphism* $\dfrac{(\ker f^*)^\perp}{\ker f^*} \cong \ker \Theta_{f^* \mathrm{J}(Y)} = \ker \Theta_{\mathrm{P}(f)} = f^* \mathrm{J}(Y) \cap \mathrm{P}(f)$.
- *The kernel of the isogeny* $\mu : \mathrm{J}(Y) \times \mathrm{P}(f) \to \mathrm{J}(X) : (y, p) \mapsto f^* y + p$ *is isomorphic to* $(\ker f^*)^\perp$.

Prym variety of a Galois covering

Notation

For the Galois cover $\tilde{f} : \tilde{X} \to Y$ the group G acts on $J(\tilde{X})$. So, there is a homomorphism

$$\mathbb{Q}[G] \to \operatorname{End}_{\mathbb{Q}} J(\tilde{X}) = \operatorname{End} J(\tilde{X}) \otimes \mathbb{Q}.$$

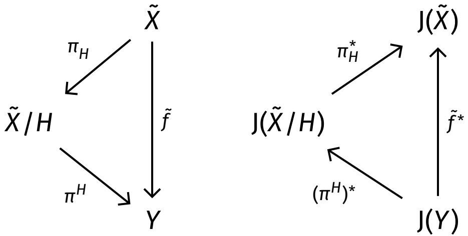
Under this identification, the *norm endomorphism* $\mathrm{Nm}(G)$ is defined by:

$$\mathrm{Nm}(G) = \sum_{g \in G} g.$$

For any subgroup H of G, we denote by $J(\tilde{X})^H$ the set of fixed points of H.

Remark

- $f^* J(Y) = \left(J(\tilde{X})^G\right)^0$
- $P(f) = \left(\ker \mathrm{Nm}(G)\right)^0$
- $\pi_H^* P(\pi^H) = J(\tilde{X})^H \cap \ker \sum_{g \in G/H} g$



Isotypical and Group Algebra Decomposition

Isotypical decomposition of $J(\tilde{X})$

Let
$$\mathbb{Q}[G] = \mathbb{Q}[G]e_1 \oplus \cdots \oplus \mathbb{Q}[G]e_r$$
be the unique decomposition of $\mathbb{Q}[G]$ into simple $\mathbb{Q}$-algebras.
Let $\operatorname{im} e_i$ be the image of e_i as an element of $\operatorname{End}_{\mathbb{Q}} J(\tilde{X})$. Then the addition map

$$\mu : \operatorname{im} e_1 \times \ldots \times \operatorname{im} e_n \to J(\tilde{X})$$

is a G-equivariant isogeny called *isotypical decomposition* of $J(\tilde{X})$.

Group algebra decomposition of $J(\tilde{X})$

For each e_i, there is a decomposition

$$e_i = b_i^1 + \ldots + b_i^{n_i}$$

into primitive orthogonal idempotents, which induces a decomposition

$$\operatorname{im} b_i^1 \times \cdots \times \operatorname{im} b_i^{n_i} \to \operatorname{im} e_i.$$

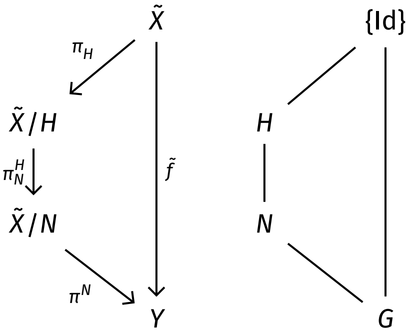
We have $\operatorname{im} b_i^j \sim \operatorname{im} b_i^k$; set $B_i := \operatorname{im} b_i^1$, then

$$B_1^{n_1} \times \ldots \times B_r^{n_r} \sim J(\tilde{X}).$$

Decomposition by Prym Varieties

Notation

Let H and N be subgroups of G such that $H \subset N$. Denote:



Theorem (Carocca and Rodríguez (2006))

Let $V_j \in \mathrm{Irr}_{\mathbb{C}}\, G$, of Schur index m_j, be the representation associated to e_j. Then

$$P(\pi_N^H) \sim B_2^{s_2} \times \cdots \times B_r^{s_r},$$

where $s_j = \dfrac{\left(\langle \mathrm{Ind}_H^G 1_H, V_j\rangle - \langle \mathrm{Ind}_N^G 1_N, V_j\rangle\right)}{m_j}.$

In particular, if $\mathrm{Ind}_H^G 1_H = W \oplus \mathrm{Ind}_N^G 1_N$ *for* $W \in \mathrm{Irr}_{\mathbb{Q}}\, G$*, then*

$$P(\pi_N^H) \sim B_W.$$

Section 2

Motivation and Problems

Motivation and Problems

Some works on Prym varieties and the decomposition of Jacobians with a group action:

- Mumford (1974) studied the classical Prym varieties of degree 2 coverings.
- Recillas and Rodríguez (1998, 2003) studied the isotypical decomposition of Galois closures of degree 3 and 4, respectively.
- Sánchez-Argáez (1999) and Carocca et al. (2002) studied the isotypical decomposition of the Jacobian of a curve with an A_5 and a dihedral action, respectively.
- Carocca and Rodríguez (2006) studied the general theory for the group algebra decomposition.

Problems

Given a degree 5 covering map $f : X \to Y$, the problems are two:

1. To determine, *as precisely as possible,* its Galois closure in terms of the ramification data.
2. To give the isotypical and group algebra decomposition of its Galois closure in terms of Prym varieties.

Section 3

Current Results

Transitive Groups of Degree 5

Possible monodromy groups

Since the monodromy group G is a transitive subgroup of S_5, we list them modulo isomorphism:

- $C_5 \cong \langle(1\ 2\ 3\ 4\ 5)\rangle$
- $D_5 \cong \langle(1\ 2\ 3\ 4\ 5), (2\ 5)(3\ 4)\rangle$
- $GA(1,5) \cong \langle(1\ 2\ 3\ 4\ 5), (2\ 3\ 5\ 4)\rangle$
- $A_5 \cong \langle(1\ 2\ 3\ 4\ 5), (1\ 3\ 4\ 5\ 2)\rangle$
- $S_5 \cong \langle(1\ 2\ 3\ 4\ 5), (1\ 2)\rangle$

G	Cycle structures
C_5	[5]
D_5	[5] or [2, 2, 1]
$GA(1,5)$	[5], [4, 1] or [2, 2, 1]
A_5	[5], [3, 1, 1] or [2, 2, 1]
S_5	[5], [4, 1], [3, 2], [3, 1, 1], [2, 2, 1] or [2, 1, 1, 1]

Cycle structure of permutations in G.

Monodromy Group for $g \geq 1$

Definition

A tuple $(t_1 \dots, t_n)$ of ramification types with an even amount of odd values is called *realizable.*

Theorem

Let g be the genus of Y and assume $g \geq 1$. Then:

- *Suppose $B = \emptyset$. If $g = 1$, then $G \cong C_5$. If $g > 1$, then G may be any transitive subgroup of S_5.*
- *If there is at least one branch value of type $[3,2]$ or $[2,1,1,1]$, then $G \cong S_5$.*
- *If there are branch value of types $[3,1,1]$ and $[4,1]$, then $G \cong S_5$.*
- *If there are no branch values of type $[3,2]$, $[3,1,1]$ or $[2,1,1,1]$; but there are of type $[4,1]$, then $G \cong S_5$ or $G \cong GA(1,5)$.*
- *If there are no branch values of type $[4,1]$, $[3,2]$ or $[2,1,1,1]$; but there are of type $[3,1,1]$, then $G \cong S_5$ or $G \cong A_5$.*

Monodromy Group for $g \geq 1$

Theorem (Continuation)

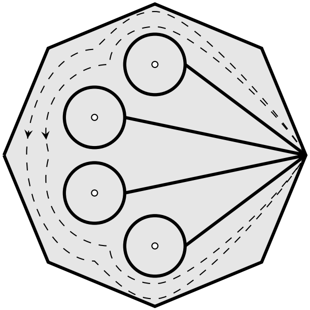
- *Suppose there no branch values of type* $[4,1]$, $[3,2]$, $[3,1,1]$ *or* $[2,1,1,1]$*; but there are branch values of type* $[2,2,1]$*. There are three cases:*
    - *If there is just one branch value and* $g = 1$*; then* $G \cong S_5$*.*
    - *If there are more than one branch value, an odd amount of them are of type* $[2,2,1]$*, and* $g > 1$*; then* $G \cong S_5$ *or* $G \cong A_5$*.*
    - *If there is an even number of branch values of type* $[2,2,1]$*, then* G *may be any listed group but* C_5*.*
- *Suppose every branch value is of type* $[5]$*. There are two cases:*
    - *If it is just one, then* G *may be ant listed group but* C_5*.*
    - *If there are more than one, then* G *may be any of the listed groups.*

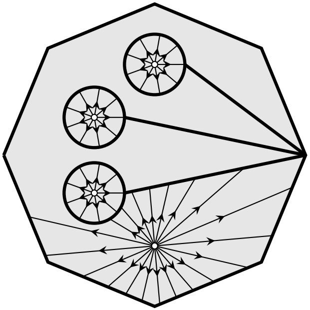
Conversely, for each realizable types of branch values tuple and for every possible monodromy group previously stated, there is a holomorphic map with that types of branch values and that monodromy group.

Ideas of the Proof

- We compute the fundamental group of $Y - B$:

$$\pi_1(Y-B) = \begin{cases} \left\langle \alpha_1,\beta_1,\dots,\alpha_g,\beta_g : \prod_{i=1}^{g_Y}[\alpha_i,\beta_i]=1 \right\rangle & n=0, \\ \left\langle \alpha_1,\beta_1,\dots,\alpha_g,\beta_g,\gamma_1,\dots,\gamma_n : \prod_{i=1}^{g_Y}[\alpha_i,\beta_i]=\prod_{i=1}^{n}\gamma_i \right\rangle & n>0. \end{cases}$$





- We use the lemma on small loops to discard cases according to the cycle structure of permutations in each possible G.
- For the existence part of the theorem, we give explicit surjective homomorphisms $\pi_1(Y - B) \to G \subset S_5$ that, trough the monodromy correspondence, define the maps f.
- For the case where $g > 2$ we have no other restriction because each listed group can be generated by two elements.
- For discarding some of the remaining cases we prove some auxiliary lemmas, for example that $\prod_{j=1}^{n} \rho(\gamma_j) \in G^{(1)}$.

Ramification Divisor in Terms of Monodromy

Corollary

If $g \geq 1$, then:

- *For $G = C_5$, $R_f = \sum_{j=1}^{n_1} 4p_j$ with $n \geq 1$.*
- *For $G = D_5$, $R_f = \sum_{j=1}^{n_1} 4p_j + \sum_{j=1}^{n_2}(q_j + r_j)$ with even n_2 and $f(q_j) = f(r_j)$. If $g = 1$, then $n_1 + n_2 > 0$.*
- *For $G = GA(1,5)$, $R_f = \sum_{j=1}^{n_1} 4p_j + \sum_{j=1}^{n_2}(q_j + r_j) + \sum_{j=1}^{n_3} 3s_j$ with even n_3 and $f(q_j) = f(r_j)$. If $g = 1$, then $n_1 + n_2 + n_3 > 0$.*
- *For $G = A_5$, $R_f = \sum_{j=1}^{n_1} 4p_j + \sum_{j=1}^{n_2}(q_j + r_j) + \sum_{j=1}^{n_4} 2t_j$ with $f(q_j) = f(r_j)$. If $g = 1$, then $n_1 + n_2 + n_4 > 0$. Moreover, if $g = 1$ and $n_1 = n_4 = 0$, then $n_2 \geq 2$.*
- *For $G = S_5$, $R_f = \sum_{j=1}^{n_1} 4p_j + \sum_{j=1}^{n_2}(q_j + r_j) + \sum_{j=1}^{n_3} 3s_j + \sum_{j=1}^{n_4} 2t_j + \sum_{j=1}^{n_5}(2u_j + v_j) + \sum_{j=1}^{n_6} w_j$ with even n_3, n_5 and n_6; $f(q_j) = f(r_j)$; $f(u_j) = f(v_j)$; $f(s_j) \neq f(w_k)$; and $f(w_j) \neq f(w_k)$ for $j \neq k$. If $g = 1$, then $\sum n_i > 0$.*

And for $g = 0$?

Theorem

If $g = 0$, then:

- *For $G = C_5$, $R_f = \sum_{j=1}^{n_1} 4p_j$ with $n \geq 2$.*
- *For $G = D_5$, $R_f = \sum_{j=1}^{n_1} 4p_j + \sum_{j=1}^{n_2}(q_j + r_j)$ with even n_2 and $f(q_j) = f(r_j)$. If $n_1 = 0$, then $n_2 \geq 4$.*
- *For $G = GA(1,5)$, $R_f = \sum_{j=1}^{n_1} 4p_j + \sum_{j=1}^{n_2}(q_j + r_j) + \sum_{j=1}^{n_3} 3s_j$ with even $n_3 \geq 2$ and $f(q_j) = f(r_j)$. If $n_3 = 2$, then $n_1 + n_2 > 0$.*
- *For $G = A_5$, $R_f = \sum_{j=1}^{n_1} 4p_j + \sum_{j=1}^{n_2}(q_j + r_j) + \sum_{j=1}^{n_4} 2t_j$ with $f(q_j) = f(r_j)$. If $n_1 = 2$, then $n_2 + n_4 > 0$. If $n_1 = 1$ and $n_2 = 0$, then $n_4 \geq 3$. If $n_1 = 0$, then $n_2 + n_3 \geq 4$.*
- *The computations for $G = S_5$ are work in progress.*

Isotypical Decomposition for $G \cong C_5$

Theorem

If $G \cong C_5$, then f is its own Galois closure. Its ramification divisor is of the form $R_f = \sum_{j=1}^{n} 4p_j$. We have

$$\dim J(X) = 5g - 4 + 2n \qquad \text{and} \qquad \dim P(f) = 4g - 4 + 2n.$$

The isogeny

$$\mu : J(Y) \times P(f) \to J(X) : (y, p) \mapsto f^*y + p$$

is G-invariant and yields the isotypical (and group algebra) decomposition of $J(X)$ up to isogeny. Moreover,

$$|\ker \mu| = \begin{cases} 5^{2g} & n \geq 2, \\ 5^{2g-1} & n = 0, \end{cases} \quad \operatorname{type}(\Theta_{f^* J(Y)}) = \begin{cases} (\underbrace{5, \dots, 5}_{g}) & n \geq 2, \\ (1, \underbrace{5, \dots, 5}_{g-1}) & n = 0, \end{cases} \quad \operatorname{type}(\Theta_{P(f)}) = \begin{cases} (\underbrace{1, \dots, 1}_{3g-4+2n}, \underbrace{5, \dots, 5}_{g}) & n \geq 2, \\ (\underbrace{1, \dots, 1}_{3g-3+2n}, \underbrace{5, \dots, 5}_{g-1}) & n = 0. \end{cases}$$

Proof

- We already know the ramification divisor. The dimension of $J(X)$ and $P(f)$ are computed trough Riemman-Hurwitz formula.
- We have

$$\mathrm{Ind}_{\{\mathrm{Id}\}}^{C_5} 1 = 1 \oplus \rho,$$

  where $\rho \in \mathrm{Irr}_{\mathbb{Q}} C_5$ is the only nontrivial irreducible rational representation of C_5. So, the action of G on $P(f)$ is a multiple of ρ. This yields the assertion about μ.
- Its kernel is computed using that f^* is injective if and only if it doesn't factor trough an cyclic unramified covering. The type of the induced polarization is computed trough the same fact and considering that their kernels are contained in $P(f)$[5].

Isotypical Decomposition for $G \cong \mathrm{D}_5$

Theorem

Suppose $G \cong \mathrm{D}_5 = \langle r, s : r^5 = s^2 = rsrs = 1 \rangle$*. Then, the signature of* $\tilde{f}$ *is* $(g; \underbrace{2, \dots, 2}_{n_2}, \underbrace{5, \dots, 5}_{n_1})$ *and:*

$$\dim \mathrm{P}(\pi^{\langle s \rangle}) = 4g - 4 + 2n_1 + n_2, \quad \dim \mathrm{P}(\pi^{\langle r \rangle}) = g - 1 + n_2/2,$$

$$\dim \mathrm{P}(\pi_{\langle r \rangle}) = 8g - 8 + 4n_1 + 2n_2 \quad \textit{and} \quad \mathrm{J}(\tilde{X}) = 10g - 9 + 4n_1 + 5n_2/2.$$

The isogenies

$$\mu_1 : \ \mathrm{J}(Y) \times \mathrm{P}(\pi^{\langle r \rangle}) \times \mathrm{P}(\pi_{\langle r \rangle}) \to \mathrm{J}(\tilde{X}) : (y, p, q) \mapsto \tilde{f}^* y + \pi^*_{\langle r \rangle} p + q$$

$$\mu_2 : \ \mathrm{J}(Y) \times \mathrm{P}(\pi^{\langle r \rangle}) \times \mathrm{P}(\pi^{\langle s \rangle})^2 \to \mathrm{J}(\tilde{X}) : (y, p, q, t) \mapsto \tilde{f}^* y + \pi^*_{\langle r \rangle} p + \pi^*_{\langle s \rangle} q + r\,\pi^*_{\langle s \rangle} t$$

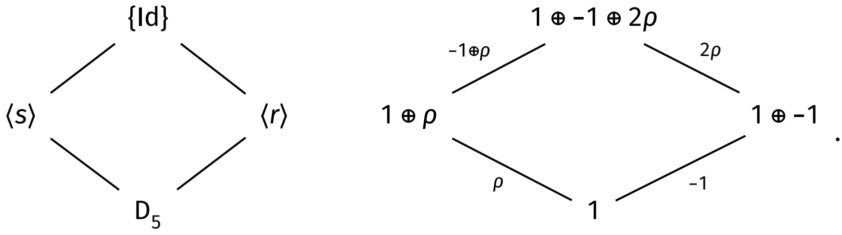
are G-invariant and yield the isotypical and group algebra decomposition of $\mathrm{J}(X)$*, respectively.*

Proof

- The signature is easily computed because the cyclic subgroups of D_5 are of order 2 or 5. The dimension of the Jacobian and Prym varieties follows from the signature.
- Let -1 and ρ be the irreducible rational representations of D_5 with the following traces:

$$\chi_{-1} : 1 \mapsto 1, r \mapsto 1, s \mapsto -1,$$
$$\chi_{\rho} : 1 \mapsto 4, r \mapsto -1, s \mapsto 0.$$

We have the following representations of D_5 induced by trivial representations of their subgroups:



- The differences between those induced representations yield the isogenies μ_1 and μ_2. Moreover, it can be explicitly computed that the action of G in $P(\pi^{\langle s\rangle})^2$ is given by

$$s \mapsto \begin{pmatrix} 0 & -1 \\ 1 & r + r^4 \end{pmatrix} \qquad r \mapsto \begin{pmatrix} 1 & r + r^4 \\ 0 & -1 \end{pmatrix}.$$

- The kernels of μ_1 and μ_2, as also the type of the induced polarizations, have not yet been computed.

Isotypical Decomposition for $G \cong \mathrm{GA}(1,5)$

Theorem

Suppose $G \cong \mathrm{GA}(1,5) = \langle r, m : r^5 = s^4 = mrm^{-1} = r^2 \rangle$. *Then, the signature of its Galois closure* $\tilde{f}$ *is* $(g; \underbrace{2,\dots,2}_{n_2}, \underbrace{4,\dots,4}_{n_3}, \underbrace{5,\dots,5}_{n_1})$ *and:*

$$\dim \mathrm{P}\big(\pi^{\langle m \rangle}\big) = 4g - 4 + 2n_1 + n_2 + 3n_3/2, \quad \dim \mathrm{P}\big(\pi^{\langle r,m^2 \rangle}\big) = g - 1 + n_3/2,$$

$$\dim \mathrm{P}\big(\pi^{\langle r \rangle}_{\langle r,m^2 \rangle}\big) = 2g - 2 + n_2 + n_3 \quad \text{and} \quad \mathrm{J}(\tilde{X}) = 20g - 19 + 8n_1 + 5n_2 + 15n_3/2.$$

The isotypical and group algebra decomposition are respectively given by:

$$\mathrm{J}(\tilde{X}) \sim \mathrm{J}(Y) \times \mathrm{P}\big(\pi^{\langle r,m^2 \rangle}\big) \times \mathrm{P}\big(\pi^{\langle r \rangle}_{\langle r,m^2 \rangle}\big) \times \mathrm{P}\big(\pi_{\langle r \rangle}\big),$$

$$\mathrm{J}(\tilde{X}) \sim \mathrm{J}(Y) \times \mathrm{P}\big(\pi^{\langle r,m^2 \rangle}\big) \times \mathrm{P}\big(\pi^{\langle r \rangle}_{\langle r,m^2 \rangle}\big) \times \mathrm{P}\big(\pi^{\langle m \rangle}\big)^4.$$

Proof

- In this case, we have to be careful in the computation of the signature because, a priori, the preimages by $\pi_{\langle m\rangle}$ of a branch value of type $[2,2,1]$ can have an stabilizer of order 2 or 4. We have that

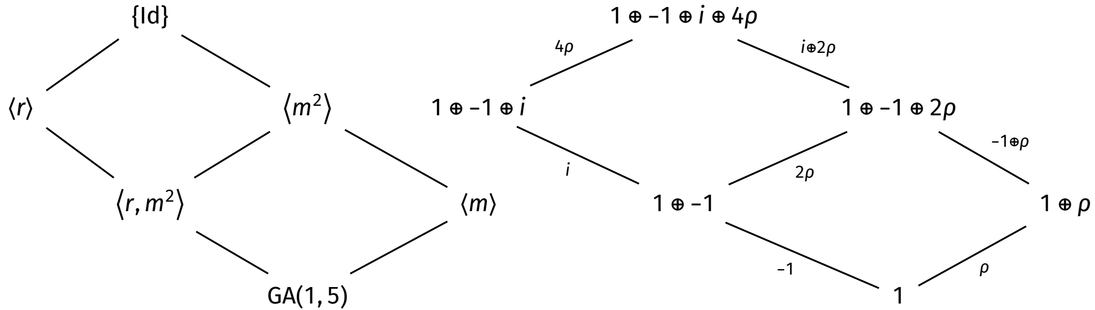
$$2 \cdot |\langle m\rangle \cap \operatorname{Stab} x| = |\operatorname{Stab} x|$$

where x is any pre-image by $\pi_{\langle m\rangle}$ of branch point of f multiplicity 2. If $|\operatorname{Stab} x| = 4$, then $|\langle m\rangle \cap \operatorname{Stab} x| = 2$. But two subgroups of order 4 of $\mathrm{GA}(1,5)$ cannot intersect in just 2 elements. Therefore, $|\operatorname{Stab} x| = 2$.

- The dimensions of the Jacobian and Prym varieties follows directly from the signature.
- Denote the characters of the irreducible rational representations of $\mathrm{GA}(1,5)$ by:

$$\begin{aligned}
\chi_{-1} &: 1 \mapsto 1, r \mapsto 1, m \mapsto -1, m^2 \mapsto 1,\\
\chi_i &: 1 \mapsto 2, r \mapsto 2, m \mapsto 0, m^2 \mapsto -2,\\
\chi_\rho &: 1 \mapsto 4, r \mapsto -1, m \mapsto 0, m^2 \mapsto 0.
\end{aligned}$$

We have the following representations of D_5 induced by trivial representations of their subgroups:



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The differences between those induced representations yield the isogenies.
A concrete isogeny with the types of the induced polarizations have not yet been computed. The cases where G is the alternating group A_5 or the whole symmetric group S_5 are work in progress.

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