ISOMETRIES OF IDEAL LATTICES AND HYPERKÄHLER MANIFOLDS

SAMUEL BOISSIÈRE, CHIARA CAMERE, GIOVANNI MONGARDI, AND ALESSANDRA SARTI

Abstract. We prove that there exists a holomorphic symplectic manifold deformation equivalent to the Hilbert scheme of two points on a K3 surface that admits a non-symplectic automorphism of order 23, that is the maximal possible prime order in this deformation family. The proof uses the theory of ideal lattices in cyclomotic fields.

1. Introduction

The study of automorphisms on deformation families of hyperkähler manifolds is a recent and very active field of research. One of the main objectives in the recent published papers concerns the classification of prime order automorphisms: fixed locus, moduli spaces and deformations. We refer for instance to [7, 15, 19] and references therein for a more complete picture. The purpose of this paper is to answer a question of [8] concerning the existence of automorphisms of order 23.

Let X be an irreducible holomorphic symplectic manifold. Its second cohomology group $H^2(X,\mathbb{Z})$ is an integral lattice for the Beauville–Bogomolov–Fujiki quadratic form [6]. Let f be a holomorphic automorphism of X of prime order p acting non-symplectically: f acts on $H^{2,0}(X)$ by multiplication by a primitive p-th root of the unity. Such automorphisms can exist only when X is projective (see [5, Proposition 6]). It follows that the invariant lattice $\mathrm{T}(f) \subset H^2(X,\mathbb{Z})$ is a primitive sublattice of the Néron–Severi group $\mathrm{NS}(X)$ and consequently the characteristic polynomial of the action of f on the transcendental lattice $\mathrm{Trans}(X)$ is the k-th power of the p-th cyclotomic polynomial Φ_p. Thus $k\varphi(p) = \mathrm{rank}_{\mathbb{Z}}(\mathrm{Trans}(X))$ and in particular

$$\varphi(p) \leq b_2(X) - \rho(X),$$

where φ is the Euler's totient function and $\rho(X) = \mathrm{rank}_{\mathbb{Z}}\,\mathrm{NS}(X)$ is the Picard number of X. Assume that X is in the deformation class of the Hilbert scheme of two points on a projective K3 surface (an IHS-$K3^{[2]}$ for short). Since $b_2(X) = 23$, the maximal order for f is $p = 23$ and this can happen only when $\rho(X) = 1$. The main result of this paper is:

Theorem 1.1. *There exists a unique IHS-$K3^{[2]}$ with a non-symplectic automorphism of order* 23.

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Observe that we do not claim unicity of the automorphism. We show in §3 that the Néron–Severi group of this variety has rank one, generated by an ample line bundle of square 46 with respect to the Beauville–Bogomolov–Fujiki quadratic form: up to now there does not exist any geometric construction of such an IHS-$K3^{[2]}$ (see [18]). We emphasize that such an automorphism can not exist on the Hilbert scheme of two points on a K3 surface since this has Picard number two.

The strategy of the proof consists in constructing an isometry of order 23 of the lattice $E_8^{\oplus 2} \oplus U^{\oplus 3} \oplus \langle -2\rangle$ with the required properties (Corollary 5.4) and then to use the surjectivity of the period map and the global Torelli theorem to construct the variety with its automorphism (Theorem 6.1). Assuming that such an automorphism does exist, the invariant lattice T and its orthogonal complement S are uniquely determined up to isometry so the main step (Proposition 5.3) consists in constructing an order 23 isometry on the lattice S: we obtain it by proving that the lattice S can be realized as an ideal lattice in the 23rd cyclotomic field, using results of Bayer–Fluckiger [2, 3, 4].

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2. Preliminaries on lattices

A *lattice* L is a free $\mathbb{Z}$-module equipped with a nondegenerate symmetric bilinear form $\langle \cdot, \cdot \rangle_L$ with integer values. Its *dual lattice* is $L^\vee := \operatorname{Hom}_{\mathbb{Z}}(L, \mathbb{Z})$. It can be also described as follows:

$$L^\vee \cong \{x \in L \otimes \mathbb{Q} \mid \langle x, v\rangle_L \in \mathbb{Z} \quad \forall v \in L\}.$$

Clearly L is a sublattice of $L^\vee$ of the same rank, so the *discriminant group* $A_L := L^\vee/L$ is a finite abelian group whose order is denoted d_L and called the *discriminant of* L. In a basis $(e_i)_i$ of L, for the Gram matrix $M := (\langle e_i, e_j\rangle_L)_{i,j}$ one has $d_L = |\det(M)|$.

A lattice L is called *even* if $\langle x, x\rangle \in 2\mathbb{Z}$ for all $x \in L$. In this case the bilinear form induces a quadratic form $q_L : A_L \longrightarrow \mathbb{Q}/2\mathbb{Z}$. Denoting by $(s_{(+)}, s_{(-)})$ the signature of $L \otimes \mathbb{R}$, the triple of invariants $(s_{(+)}, s_{(-)}, q_L)$ characterizes the *genus* of the even lattice L (see [7, 11] and references therein).

A lattice L is called *unimodular* if $A_L = \{0\}$. A sublattice $M \subset L$ is called *primitive* if L/M is a free $\mathbb{Z}$-module. If L is unimodular and $M \subset L$ is a primitive sublattice, then M and its orthogonal $M^\perp$ in L have isomorphic discriminant groups and $q_M = -q_{M^\perp}$ (see [1, § I.2]).

Let p be a prime number. A lattice L is called *p-elementary* if $A_L \cong \left(\frac{\mathbb{Z}}{p\mathbb{Z}}\right)^{\oplus a}$ for some non negative integer a (also called the *length* $\ell(A_L)$ of A). We write $\frac{\mathbb{Z}}{p\mathbb{Z}}(\alpha)$, $\alpha \in \mathbb{Q}/2\mathbb{Z}$, to denote that the quadratic form q_L takes value α on the generator of the $\frac{\mathbb{Z}}{p\mathbb{Z}}$ component of the discriminant group. Recall that an even indefinite p-elementary lattice of rank $r \geq 3$ with $p \geq 3$ is uniquely determined by its signature and discriminant form (see [7, Theorem 2.2]).

3. Basic results on non-symplectic automorphisms

From now on, we assume that X is an IHS-$K3^{[2]}$ with a non-symplectic automorphism f of prime order $3 \leq p \leq 23$. The lattice $H^2(X, \mathbb{Z})$ has signature $(3, 20)$ and is isometric to $L := E_8^{\oplus 2} \oplus U^{\oplus 3} \oplus \langle -2\rangle$, where U is the unique even unimodular hyperbolic lattice of rank two and E_8 is the negative definite lattice associated to the corresponding Dynkin diagram. We restate in this special case some results of Boissière–Nieper-Wisskirchen–Sarti [8]: the case $p = 23$ was left apart since it requires different arguments due to the fact that the ring of integers of the 23rd cyclotomic field is not a PID, but some basic facts extend easily.

The automorphism f induces an isometry $g := f^*$ on $H^2(X, \mathbb{Z})$. We denote by $G = \langle g \rangle$ the group generated by g and we put

$$\tau := g - 1 \in \mathbb{Z}[G], \quad \sigma := 1 + g + \cdots + g^{p-1} \in \mathbb{Z}[G].$$

One has $\mathrm{T}(f) = \ker(\tau) \cap H^2(X, \mathbb{Z})$ and we define $\mathrm{S}(f) := \ker(\sigma) \cap H^2(X, \mathbb{Z})$.

Denote by $\Phi_p := 1 + T + \cdots + T^{p-1} \in \mathbb{Q}[T]$ the p-th cyclotomic polynomial. Consider the cyclotomic field $K = \mathbb{Q}[T]/(\Phi_p) \cong \mathbb{Q}(\zeta_p)$ with ring of algebraic integers $O_K \cong \mathbb{Z}[\zeta_p]$, where ζ_p denotes the class of T modulo Φ_p. The G-module structure of K is defined by $g \cdot x = \zeta_p x$ for $x \in K$. For any fractional ideal I in K, and $\alpha \in I$, we denote by (I, α) the module $I \oplus \mathbb{Z}$ whose G-module structure is defined by $g \cdot (x, k) = (\zeta_p x + k\alpha, k)$. By a theorem of Diederichsen–Reiner [10, Theorem 74.3], $H^2(X, \mathbb{Z})$ is isomorphic as a $\mathbb{Z}[G]$-module to a direct sum:

$$(A_1, a_1) \oplus \cdots \oplus (A_r, a_r) \oplus A_{r+1} \oplus \cdots A_{r+s} \oplus Y \tag{1}$$

for some $r, s \in \mathbb{N}$, where the A_i are fractional ideal in K, $a_i \in A_i$ are such that $a_i \notin (\zeta_p - 1)A_i$ and Y is a free $\mathbb{Z}$-module of finite rank on which G acts trivially.

Lemma 3.1. *The quotient* $\frac{H^2(X,\mathbb{Z})}{\mathrm{T}(f) \oplus \mathrm{S}(f)}$ *is a p-torsion module.*

Proof. First we observe that $\mathrm{T}(f) \cap \mathrm{S}(f) = 0$ since $H^2(X, \mathbb{Z})$ has no p-torsion. We want to show that for any $x \in H^2(X, \mathbb{Z})$, one has $px \in \mathrm{T}(f) \oplus \mathrm{S}(f)$. We consider the direct sum decomposition (1), where by abuse of notation, we identify $\mathrm{T}(f)$ and $\mathrm{S}(f)$ with their images. It is clear that $Y \subset \mathrm{T}(f)$ and $O_K \subset \mathrm{S}(f)$. Let $A = \sum_i O_K \alpha_i$, with $\alpha_i \in K$, be one of the fractional ideals of K that occur in the decomposition of $H^2(X, \mathbb{Z})$. Clearly $A \subset \mathrm{S}(f)$. Consider a term $(A, a) = A \oplus \mathbb{Z}$ of the decomposition of $H^2(X, \mathbb{Z})$. To show that $pv \subset \mathrm{T}(f) \oplus \mathrm{S}(f)$ for any $v \in (A, a)$ it is enough to consider the case $v = (0, 1)$. One has $\tau(pv) = (pa, 0)$. Write $a = \sum_i x_i \alpha_i$ with $x_i \in O_K$. Since $O_K/(\zeta_p - 1) \cong \mathbb{Z}/p\mathbb{Z}$, there exists $z_i \in O_K$ such that $px_i = (\zeta_p - 1)z_i$. Hence $\tau(pv) = ((\zeta_p - 1)z, 0)$ with $z := \sum_i z_i \alpha_i \in A$. Now $\tau((z, 0)) = ((\zeta_p - 1)z, 0)$, hence $\tau((pv - (z, 0)) = 0$ and $\sigma((z, 0)) = 0$, so finally $pv = (pv - (z, 0)) + (z, 0) \in \mathrm{T}(f) \oplus \mathrm{S}(f)$. □

By Lemma 3.1 there exists a non-negative integer a_f such that

$$\frac{H^2(X, \mathbb{Z})}{\mathrm{T}(f) \oplus \mathrm{S}(f)} \cong \left(\frac{\mathbb{Z}}{p\mathbb{Z}}\right)^{\oplus a_f}.$$

By definition, $\mathrm{S}(f)$ is a torsion-free O_K-module for the action $\zeta_p \cdot x = g(x)$ for all $x \in S(f)$, hence $\mathrm{S}(f)_{\mathbb{Q}} := \mathrm{S}(f) \otimes_{\mathbb{Z}} \mathbb{Q}$ is a K-vector space. It follows that there exists a positive integer m_f such that

$$\mathrm{rank}_{\mathbb{Z}} \mathrm{S}(f) = \dim_{\mathbb{Q}} \mathrm{S}(f)_{\mathbb{Q}} = (p - 1)m_f.$$

It is easy to check that $\mathrm{S}(f)$ is the orthogonal complement of $\mathrm{T}(f)$ in the lattice $H^2(X, \mathbb{Z})$ (see [8, Lemma 6.1]). By a similar argument as in [8, Lemma 6.5], one deduces from Lemma 3.1 that the invariant lattice $\mathrm{T}(f)$ has signature $(1, 22 - (p - 1)m_f)$ and discriminant

$$A_{\mathrm{T}(f)} \cong \left(\frac{\mathbb{Z}}{2\mathbb{Z}}\right) \oplus \left(\frac{\mathbb{Z}}{p\mathbb{Z}}\right)^{\oplus a_f}$$

and that $\mathrm{S}(f)$ has signature $(2,(p-1)m_f-2)$ and discriminant

$$A_{\mathrm{S}(f)} \cong \left(\frac{\mathbb{Z}}{p\mathbb{Z}}\right)^{\oplus a_f}.$$

If $p \geq 3$, as explained in [8, proof of Lemma 6.5], the action of G on $A_{\mathrm{S}(f)}$ is trivial. Since f acts non-symplectically one has $\mathrm{Trans}(X) \subset \mathrm{S}(f)$ and $\mathrm{rank}_{\mathbb{Z}} \mathrm{Trans}(X) \geq p-1$. In particular, if $m_f = 1$ this forces $\mathrm{Trans}(X) = \mathrm{S}(f)$ and consequently $\mathrm{NS}(X) = \mathrm{T}(f)$. Since 1 is not an eigenvalue of $f^*|_{\mathrm{S}(f)}$ the characteristic polynomial of $f^*|_{\mathrm{S}(f)}$ is Φ_p.

All the possible isometry classes for the lattices $\mathrm{T}(f)$ and $\mathrm{S}(f)$ have been classified in [7] when $2 \leq p \leq 19$ (only partially for $p = 5$) by using the previous properties (for $p = 2$ the situation is a bit different), the Lefschetz fixed point formula and a relation between the cohomology modulo p of the fixed locus and the integers a_f, m_f obtained using Smith theory methods [8].

If $p = 23$ the only possibility is that $m_f = 1$, $\mathrm{T}(f)$ has signature $(1,0)$ and $\mathrm{S}(f)$ has signature $(2,20)$. The case $a_f = 0$ is impossible by Milnor's theorem since there exists no even unimodular lattice with signature $(2,20)$, hence $a_f = 1$ since $\mathrm{T}(f)$ has rank one, so $A_{\mathrm{T}(f)} \cong \frac{\mathbb{Z}}{46\mathbb{Z}}$ and finally $\mathrm{T}(f)$ is isometric to the lattice $\langle 46\rangle$. By [17, Theorem 1.13.5] the lattice $\mathrm{S}(f)$ splits as a direct sum $U \oplus W$ where W is hyperbolic and 23-elementary of signature $(1,19)$, $A_W \cong \frac{\mathbb{Z}}{23\mathbb{Z}}$ and by [20, §1] W is unique up to isometry. It follows that $\mathrm{S}(f)$ is uniquely determined, hence it is isometric to the lattice $U^{\oplus 2} \oplus E_8^{\oplus 2} \oplus K_{23}$ where $K_{23} := \begin{pmatrix} -12 & 1 \\ 1 & -2 \end{pmatrix}$. As a consequence, if there exists an IHS-$K3^{[2]}$, say X, with a non-symplectic automorphism f of order 23, then necessarily it has $\rho(X) = 1$, $\mathrm{NS}(X) = \mathrm{T}(f) = \langle 46\rangle$ and $\mathrm{Trans}(X) = \mathrm{S}(f) = E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23}$. Such a variety does not belong to any of the known families: Hilbert schemes of points or moduli spaces of semi-stable sheaves on projective K3 surfaces have Picard number greater than 2, Fano varieties of lines on cubic fourfolds are polarized by a class of square 6 (see [8, § 5.5.2] and references therein) and similarly the degree of the polarisation is 2 for double covers of EPW sextics, it is 38 for the sums of powers of general cubics of Iliev–Ranestad and it is 22 for the varieties of Debarre–Voisin (see [18, §0]).

4. Ideal lattices in cyclotomic fields

The relation between automorphisms of lattices with given characteristic polynomials and ideals in cyclotomic fields has been studied by many authors, in particular Bayer-Fluckiger [2, 3, 4] and Gross–McMullen [12]. We recall here some results that are needed in the sequel.

Assume that p is an odd prime number. Recall that $K = \mathbb{Q}(\zeta_p)$ denotes the cyclotomic field with ring of algebraic integers $O_K = \mathbb{Z}[\zeta_p]$. We denote respectively by $\mathrm{Tr}_{K/\mathbb{Q}}$ and $\mathrm{N}_{K/\mathbb{Q}}$ the trace and the norm maps. The complex conjugation on K is defined as the $\mathbb{Q}$-linear involution $K \to K, x \mapsto \overline{x}$ such that $\overline{\zeta_p^i} = \zeta_p^{p-i}$ for all i. We denote by $F \subset K$ the *real* subfield of K, that is

$$F := \{x \in K \mid \overline{x} = x\}.$$

Denoting $\mu_p := \zeta_p + \zeta_p^{p-1}$, one has $F = \mathbb{Q}(\mu_p)$. The $\mathbb{Q}$-linear pairing

$$(-,-)_K : K \times K \to \mathbb{Q}, \quad (x,y) \mapsto \mathrm{Tr}_{K/\mathbb{Q}}(x\overline{y})$$

is non-degenerate and has determinant $D_K := p^{p-2}$.

Let $(S, \langle -,-\rangle_S)$ be an integral even lattice of rank $p-1$, signature (s_+, s_-) and discriminant d_S. Assume that S admits a non trivial isometry φ of order p. Its characteristic polynomial is then Φ_p so S admits a natural structure of O_K-module defined by $\zeta_p \cdot x = \varphi(x)$ for all $x \in S$. For dimensional reasons $S_{\mathbb{Q}} := S \otimes_{\mathbb{Z}} \mathbb{Q}$ is isomorphic to K so the inclusion $S \hookrightarrow S_{\mathbb{Q}} \cong K$ identifies the lattice S with an O_K-submodule of K (a fractional ideal of K), so S becomes an *ideal lattice* in K. Observe that $S_{\mathbb{Q}}$ is identified with K in such a way that the isometry φ corresponds to the mutiplication by ζ_p in K. The multiplication by ζ_p is an isometry for $(-,-)_K$ and also for $\langle -,-\rangle_S$ extended to $S_{\mathbb{Q}}$ since it corresponds to the action of φ. Since the trace is non degenerate, under the identification $S_{\mathbb{Q}} = K$ there exists a unique $\alpha \in K$ such that

$$\langle x,y\rangle_S = (\alpha x, y)_K \quad \forall x,y \in S.$$

Since the bilinear form on S is symmetric one has $\overline{\alpha} = \alpha$ so $\alpha \in F$.

If $I \subset K$ is a fractional ideal and $\alpha \in F$, we denote by I_α the ideal lattice whose bilinear form is $\langle x,y\rangle_\alpha := \mathrm{Tr}(\alpha x \overline{y})$. Some of the main invariants of the lattice I_α correspond to properties of α that we explain now.

Recall that the *norm* of I is defined as $N(I) := |\det(\psi)|$ where $\psi\colon K \to K$ is any $\mathbb{Q}$-linear automorphism such that $\psi(O_K) = I$. By a direct computation one finds that the discriminant d_{I_α} of I_α satisfies the relation:

$$d_{I_\alpha} = N(I)^2 |N_{K/\mathbb{Q}}(\alpha)| D_K. \tag{2}$$

Observe that since $\alpha \in F$ one has

$$N_{K/\mathbb{Q}}(\alpha) = N_{F/\mathbb{Q}}\left(N_{K/F}(\alpha)\right) = N_{F/\mathbb{Q}}(\alpha^2) = N_{F/\mathbb{Q}}(\alpha)^2.$$

We recall that the *codifferent* $O_K^\vee$ of K is defined by

$$O_K^\vee := \{x \in K \mid \forall y \in O_K, \mathrm{Tr}_{K/\mathbb{Q}}(x\overline{y}) \in \mathbb{Z}\}.$$

If I_α is an integral lattice then for any $x,y \in I$ one has $\mathrm{Tr}_{K/\mathbb{Q}}(\alpha x \overline{y}) \in \mathbb{Z}$ so $\alpha x \overline{y} \in O_K^\vee$. The integrality of I_α is thus equivalent to the condition:

$$\alpha x \overline{y} \in O_K^\vee \quad \forall x,y \in I. \tag{3}$$

Note that if α satisfies the above property the lattice I_α is automatically even: putting $\gamma := \sum_{i=0}^{(p-1)/2} \zeta_p^i$, since $\gamma + \overline{\gamma} = 1$ one has for any $x \in I$

$$\langle x,x\rangle_S = \mathrm{Tr}_{K/\mathbb{Q}}(\alpha x\overline{x}) = \mathrm{Tr}_{K/\mathbb{Q}}((\gamma+\overline{\gamma})\alpha x \overline{x}) = \mathrm{Tr}_{K/\mathbb{Q}}(\gamma \alpha x \overline{x}) + \mathrm{Tr}_{K/\mathbb{Q}}(\overline{\gamma \alpha x \overline{x}}) \in 2\mathbb{Z},$$

since $\alpha x \overline{x} \in O_K^\vee$ by assumption.

The field K admits $p-1$ complex embeddings defined by $\zeta_p \mapsto \mathrm{e}^{\frac{2ik\pi}{p}}$, $1 \le k \le p-1$ that induce real embeddings of F. We denote by t the number of these real embeddings such that α is negative. One can show that the lattice I_α has signature

$$(p-1-2t, 2t). \tag{4}$$

This is a special case of [2, Proposition 2.2], we recall the argument for convenience. First observe that K is a quadratic extension of F with minimal polynomial $X^2 - \mu_p X + 1 \in F[X]$. Denoting $\theta := \zeta_p^2 + \zeta_p^{-2} - 2$ one has thus $K \cong F(\sqrt{\theta})$. Each complex embedding of K induces a real embedding $v\colon F \to \mathbb{R}$ such that $v(\theta) < 0$. It follows that $K \otimes_{\mathbb{Q}} \mathbb{R}$ decomposes in a direct sum

$$K \otimes_{\mathbb{Q}} \mathbb{R} = \bigoplus_{v\colon F \to \mathbb{R}} \mathbb{R}\left(\sqrt{v(\theta)}\right),$$

where the sum runs over all real embeddings of F. Each factor is isomorphic to $\mathbb{C}$ and the complex conjugation on K induces the usual complex conjugation on each factor $\mathbb{C}$. On each factor, the form $\langle -,-,\rangle_\alpha$ computed in the $\mathbb{R}$-basis $(1, \sqrt{v(\theta)})$ is $\mathrm{diag}(2v(\alpha), -2v(\alpha)v(\theta))$, so it has signature $(2,0)$ if $v(\alpha) > 0$ and signature $(0,2)$ if $v(\alpha) < 0$. The result follows.

5. Construction of isometries of lattices

We want to determine if a given integral even p-elementary lattice S of rank $p-1$ with fixed signature and discriminant form admits an isometry of order p whose characteristic polynomial is the cyclotomic polynomial Φ_p. By the results of Section 4, one first has to find an element $\alpha \in F$ satisfying conditions (2),(3),(4).

Example 5.1. Assume that $p = 5$ and $S = U \oplus H_5$ with $H_5 = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$. The lattice S is 5-elementary with $d_S = 5$, it has signature $(2,2)$ and discriminant form $A_S = \frac{\mathbb{Z}}{5\mathbb{Z}}\left(\frac{2}{5}\right)$. In [7, Table 2] this case is denoted by $(p,m,a) = (5,1,1)$. In order to recover this lattice as an ideal lattice, since O_K is a PID we take $I = \beta O_K$ for some $\beta \in K$. Equation (2) writes:

$$N_{K/\mathbb{Q}}(\beta)^2 N_{F/\mathbb{Q}}(\alpha)^2 = \frac{1}{5^2}.$$

Assuming that $\beta = 1$, we run a basic computer search program to determine $\alpha \in F$ that satisfies all the needed conditions. Taking $\alpha = \frac{1}{5}(3\mu_5 + 4)$, in the basis $(1, \zeta_5, \zeta_5^2, \zeta_5^3)$ of I the bilinear form writes

$$\begin{pmatrix} 2 & 1 & -2 & -2 \\ 1 & 2 & 1 & -2 \\ -2 & 1 & 2 & 1 \\ -2 & -2 & 1 & 2 \end{pmatrix}$$

so condition (3) is satisfied and it is easy to check that this lattice has signature $(2,2)$ and discriminant form $\frac{\mathbb{Z}}{5\mathbb{Z}}\left(\frac{2}{5}\right)$. As mentioned above, these invariants characterize the lattice $U \oplus H_5$ up to isometry. By construction, the order 5 isometry of this lattice, written in this basis, is the companion matrix of Φ_5:

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Example 5.2. Assume that $p = 13$ and that $S = U^{\oplus 2} \oplus E_8$. The lattice S is unimodular of signature $(2,10)$. In [7, Table 5] this case is denoted by $(p,m,a) = (13,1,0)$. If S admits an order 13 isometry it induces an identification $S = \beta O_K$ for some $\beta \in K$. Equation (2) writes:

$$N_{K/\mathbb{Q}}(\beta)^2 N_{F/\mathbb{Q}}(\alpha)^2 = \frac{1}{13^{11}}.$$

It is clear that this equation has no solution, so this lattice does not admit an isometry whose characteristic polynomial is Φ_{13}. This answers a question left open in [7, Theorem 7.1]: this case cannot be realized by a non-symplectic automorphism of order 13 on an IHS-$K3^{[2]}$.

We assume now that $p = 23$ and we consider the lattice $S := E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23}$. It is 23-elementary with $d_S = 23$, signature $(2, 20)$ and discriminant form $A_S = \frac{\mathbb{Z}}{23\mathbb{Z}}\left(\frac{-2}{23}\right)$.

Proposition 5.3. *The lattice $U^{\oplus 2} \oplus E_8^{\oplus 2} \oplus K_{23}$ admits an isometry of order 23 which acts trivially on the discriminant group A_S.*

Proof. We apply the strategy developed above. Taking $I = O_K$, equation (2) writes:

$$N_{F/\mathbb{Q}}(\alpha) = \frac{1}{23^{10}}.$$

The software MAGMA [9] provides only one solution to this equation:

$$\alpha_0 := \frac{1}{23}(-\mu_{23}^7 + \mu_{23}^6 + 7\mu_{23}^5 - 6\mu_{23}^4 - 14\mu_{23}^3 + 9\mu_{23}^2 + 7\mu_{23} - 2).$$

This element α_0 does not satisfy all the needed conditions, so we look for an element $\alpha = \alpha_0 \cdot \varepsilon$ with ε invertible in O_F: this element has the same norm as α_0 but letting ε vary this will produce lattices with different signature and discriminant form. By the Dirichlet unit theorem, the group of units O_F^* is the product of the finite cyclic group of roots of unity of F with a free abelian group of rank 10. A computation with the software SAGE [21] shows that $O_F^* \cong \frac{\mathbb{Z}}{2\mathbb{Z}} \times \mathbb{Z}^{10}$ (the only roots of unity in F are ± 1) where the free part is generated by the following fundamental units:

$$\begin{aligned}
\epsilon_1 &:= \mu_{23}^4 - 4\mu_{23}^2 + 2\\
\epsilon_2 &:= \mu_{23}^8 - 8\mu_{23}^6 + 20\mu_{23}^4 - 16\mu_{23}^2 + 2\\
\epsilon_3 &:= \mu_{23}^7 - 7\mu_{23}^5 + 14\mu_{23}^3 - 7\mu_{23}\\
\epsilon_4 &:= \mu_{23}^2 - 2\\
\epsilon_5 &:= \mu_{23}^9 - 9\mu_{23}^7 + 27\mu_{23}^5 - 30\mu_{23}^3 + 9\mu_{23}\\
\epsilon_6 &:= \mu_{23}^9 - 8\mu_{23}^7 + 20\mu_{23}^5 - 16\mu_{23}^3 + \mu_{23}^2 + 2\mu_{23} - 1\\
\epsilon_7 &:= \mu_{23}\\
\epsilon_8 &:= \mu_{23}^7 + \mu_{23}^6 - 6\mu_{23}^5 - 5\mu_{23}^4 + 10\mu_{23}^3 + 6\mu_{23}^2 - 4\mu_{23} - 1\\
\epsilon_9 &:= \mu_{23}^8 + \mu_{23}^7 - 8\mu_{23}^6 - 7\mu_{23}^5 + 20\mu_{23}^4 + 14\mu_{23}^3 - 16\mu_{23}^2 - 7\mu_{23} + 3\\
\epsilon_{10} &:= \mu_{23}^9 + \mu_{23}^8 - 8\mu_{23}^7 - 7\mu_{23}^6 + 20\mu_{23}^5 + 14\mu_{23}^4 - 16\mu_{23}^3 - 6\mu_{23}^2 + 3\mu_{23} - 1
\end{aligned}$$

We consider an element $\alpha \in F$ given as follows:

$$\alpha = \alpha_0 \epsilon_0 \epsilon_1^{\nu_1} \cdots \epsilon_{10}^{\nu_{10}}$$

for some $\nu_i \in \mathbb{Z}$ with $\epsilon_0 \in \{-1, 1\}$. By running a basic computer search program we find that the choice $\epsilon_0 = 1$, $\nu = (2, 1, 2, 2, 0, 1, 1, 2, 1, 0)$ satisfies all the needed conditions: in the basis $(1, \zeta_{23}, \ldots, \zeta_{23}^{21})$ of I the bilinear form is the matrix given in Appendix A. It is easy to check that this lattice has signature $(2, 20)$ and discriminant form $\frac{\mathbb{Z}}{23\mathbb{Z}}\left(\frac{44}{23}\right)$. As already mentioned, these invariants characterize the lattice $U^{\oplus 2} \oplus E_8^{\oplus 2} \oplus K_{23}$ up to isometry. By construction, the order 23 isometry of this lattice, written in this basis, is the companion matrix of the polynomial Φ_{23} and a direct computation shows that this isometry acts trivially on the discriminant group. □

Corollary 5.4. *The lattice* $L := E_8^{\oplus 2} \oplus U^{\oplus 3} \oplus \langle -2\rangle$ *admits an order* 23 *isometry whose invariant lattice is isometric to* $T := \langle 46\rangle$ *and such that the orthogonal complement of* T *in* L *is isometric to* $S := E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23}$.

Proof. Denoting $T = \mathbb{Z}t$ with $t^2 = 46$, the discriminant group A_T is generated by $\tau := t/46 \in T^\vee$ with $\tau^2 = 1/46$. We denote by $\sigma \in S^\vee$ a generator of A_S such that $\sigma^2 = 44/23$. The quadratic form takes value 8 on the vector $2\sigma + 4\tau \in (S \oplus T)^\vee$ so it is isotropic in $A_{S\oplus T}$, hence it defines an even overlattice

$$M := (S \oplus T) + (2\sigma + 4\tau)\mathbb{Z} \subset (S \oplus T)^\vee.$$

Consider the quotient $H := \frac{M}{S\oplus T} \subset A_{S\oplus T}$. The group H is totally isotropic for the finite quadratic form on $A_{S\oplus T}$. Denoting by $H^\perp$ its orthogonal complement in $A_{S\oplus T}$ one has $A_M \cong H^\perp/H$ (see [17, §5]). It is easy to check that $h := 23\tau$ generates $H^\perp/H$ and that $h^2 = 3/2 \in \mathbb{Q}/2\mathbb{Z}$. Since $2h \in T$ we conclude that M is an even lattice of signature $(3, 20)$ and discriminant form $A_M = \frac{\mathbb{Z}}{2\mathbb{Z}}\left(\frac{3}{2}\right)$. By [16, Theorem 2.2] these invariants characterize M up to isometry, so M is isomorphic to L. It follows directly from the construction that S is the orthogonal complement of T in M.

Let φ be the isometry of order 23 on S constructed in Proposition 5.3. Since φ acts trivially on A_S, the isometry $\varphi \oplus \mathrm{id}$ of $S \oplus T$ extends to an isometry on L with the required properties. □

Remark 5.5. In the lattice L, denoting by (e, f) a basis on one of the factors isometric to the lattice U and by δ a generator of the factor isometric to $\langle -2\rangle$, it is easy to see that an explicit embedding of T in L whose orthogonal complement is isometric to S is given by $t \mapsto 2e + 12f + \delta$. Applying Nikulin's results on primitive embeddings and decomposition of lattices (see [7] and references therein) one can see that T admits up to isometry a second embedding in L whose orthogonal complement is isometric to $E_8^{\oplus 2} \oplus U \oplus \langle -2\rangle \oplus \langle 2\rangle \oplus K_{23}$, an explicit embedding being given by $t \mapsto e + 23f$.

6. An IHS-$K3^{[2]}$ with a non-symplectic automorphism of order 23

Theorem 6.1. *There exists a unique IHS-*$K3^{[2]}$ *with a non-symplectic automorphism of order* 23. *This variety* X *and its automorphism* f *have the following properties:*

(1) $\rho(X) = 1$, $\mathrm{NS}(X) \cong \langle 46\rangle$ *and* $\mathrm{Trans}(X) \cong E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23}$*;*
(2) $\mathrm{T}(f) = \mathrm{NS}(X)$ *and* $\mathrm{S}(f) = \mathrm{Trans}(X)$.

Proof. The proof is an application of the surjecivity of the period map and of the global Torelli theorem for IHS manifolds.

Construction of the variety. Let $\mathcal{M}_L^0$ be a connected component of the moduli space of pairs (X, η), where X is an IHS-$K3^{[2]}$ and $\eta\colon H^2(X, \mathbb{Z}) \to L$ is an isometry. The period domain is

$$\Omega_L := \{[\omega] \in \mathbb{P}(L_\mathbb{C}) \mid \langle\omega, \omega\rangle_L = 0, \langle\omega, \overline{\omega}\rangle_L > 0\}.$$

Recall that the period map $P\colon \mathcal{M}_L^0 \to \Omega_L$ defined by $P((X, \eta)) = \eta(H^{2,0}(X))$ is surjective [13, Theorem 8.1].

Consider as in Corollary 5.4 the embedding of $T = \langle 46\rangle$ in the lattice L whose orthogonal complement is $S = E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23}$, with the isometry φ of order 23 acting trivially on T. We denote by ω a generator of the one-dimensional eigenspace of $S_{\mathbb{C}}$ corresponding to the eigenvalue $\xi := \mathrm{e}^{\frac{2i\pi}{23}}$. Recall that by construction S is identified with the ring of integers O_K of the cyclotomic field $K = \mathbb{Q}(\zeta_{23})$ so that $\omega \in S_{\mathbb{C}} = K \otimes_{\mathbb{Q}} \mathbb{C}$ with basis $(1, \zeta_{23}, \dots, \zeta_{23}^{21})$. In this basis, the isometry φ acts by the companion matrix of the 23rd cyclotomic polynomial and it is easy to check that up to a multiplicative constant one has

$$\omega = \sum_{i=0}^{21} \left(\sum_{j=0}^{21-i} \xi^j \right) \zeta_{23}^i.$$

Since $\varphi(\omega) = \xi\omega$ one has $\langle \omega, \omega\rangle_L = 0$. Denoting by $\mathrm{Tr}_{K/\mathbb{Q}} \colon K_{\mathbb{C}} \to \mathbb{C}$ the $\mathbb{C}$-linear extension of the trace, one has

$$\langle \omega, \overline{\omega}\rangle_L = \langle \omega, \overline{\omega}\rangle_S = \mathrm{Tr}_{K/\mathbb{Q}}(\omega\alpha\omega)$$

where α is given in the proof of Proposition 5.3. An explicit computation shows that $\mathrm{Tr}_{K/\mathbb{Q}}(\omega\alpha\omega) > 0$, so $\omega \in \Omega_L$. By surjectivity of the period map, there exists $(X, \eta) \in \mathcal{M}_L^0$ such that $\eta(H^{2,0}(X)) = \omega$. Then

$$\eta(\mathrm{NS}(X)) = \{\lambda \in L \mid \langle \lambda, \omega\rangle_L = 0\} \supset T.$$

Let us show that $\eta(\mathrm{NS}(X)) = T$. For this, we show that there is no element $\lambda \in S$ with the property that $\langle \lambda, \omega\rangle_S = 0$. In the basis $(1, \zeta_{23}, \dots, \zeta_{23}^{21})$, denoting $\Xi := (\xi^{21}, \dots, \xi, 1)$ and $J := \begin{pmatrix} 1 & & 1 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$ one has by definition $\omega = J\Xi$. Denote by M the matrix of the lattice S in the basis $(1, \zeta_{23}, \dots, \zeta_{23}^{21})$ (see Appendix A). For any $\lambda \in S$, since $S = O_K = \mathbb{Z}[\zeta_{23}]$ the element λ can be identified with a column vector with integer coordinates. Then

$$\langle \lambda, \omega\rangle_S = \lambda^\top M\omega = \lambda^\top MJ\Xi.$$

If $\lambda^\top MJ\Xi = 0$, since $\lambda^\top MJ$ has integer coordinates and since the coordinates of Ξ are linearly independent over $\mathbb{Q}$ it follows that $\lambda^\top MJ = 0$. But the matrix MJ is invertible, so $\lambda = 0$. This proves that $\mathrm{NS}(X) \cong T$ and in particular X is projective [13, Theorem 3.11]. Since $\mathrm{NS}(X) = \langle 46\rangle$, it follows from [14, Theorem 2.2] that the fiber $P^{-1}(\omega)$ is a singleton so the marked variety (X, η) is unique.

Construction of the automorphism. The isometry φ preserves $H^{2,0}(X) = \mathbb{C}\omega$ so it is a Hodge isometry. Denoting by q_X the Beauville-Bogomolov-Fujiki quadratic form on $H^2(X, \mathbb{Z})$, the positive cone $\widetilde{C}_X$ in $H^2(X, \mathbb{R})$ has a distinguished orientation given by the connected component C_X of the cone

$$\{x \in H^{1,1}(X) \cap H^2(X, \mathbb{R}) \mid q_X(x) > 0\}$$

that contains the Kähler cone (see [14, § 4]). By Markman [14, Lemma 9.2] the group of monodromy operators of $H^2(X, \mathbb{Z})$ is equal to the group of isometries of $H^2(X, \mathbb{Z})$ preserving the orientation of $\widetilde{C}_X$, or equivalently leaving invariant the positive cone C_X. Here the generator of $\mathrm{NS}(X) \cong T$ is an ample class so it lives in the Kähler cone and since $\mathrm{NS}(X)$ is invariant by φ the cone C_X is preserved, so φ is a monodromy operator that leaves invariant a Kähler class. By the Global Torelli Theorem of Markman–Verbitsky [14, Theorem 1.3] there exists an automorphism f

of X such that $f^* = \varphi$ on $H^2(X,\mathbb{Z})$. Since the natural map $\mathrm{Aut}(X) \to O(H^2(X,\mathbb{Z}))$ is injective (see for instance [15, Lemma 1.2] and references therein), f is an order 23 non-symplectic automorphism of X. $\square$

Remark 6.2. The variety X with its automorphism of order 23 is uniquely determined inside a 20-dimensional family of IHS-$K3^{[2]}$ polarized by a class of square 46. However, the automorphism of order 23 might not be unique. The same method can be used to produce order 23 automorphisms on deformations of $K3^{[n]}$ with $n \geq 3$, under some arithmetic conditions on n.

Appendix A. Matrix of the lattice with an order 23 isometry

Here is the matrix of the bilinear form on the lattice $U^{\oplus 2} \oplus E_8^{\oplus 2} \oplus K_{23}$, written in a basis such that the order 23 isometry of this lattice is the companion matrix of the cyclotomic polynomial Φ_{23}:

$$\begin{pmatrix}
-2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 \\
3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 \\
0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 \\
3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 \\
0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 \\
2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 \\
1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 \\
0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 \\
-1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 \\
-2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 \\
-2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 \\
-3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 \\
-3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 \\
-2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 \\
-2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 \\
-1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 \\
0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 \\
1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 \\
2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 \\
0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 \\
3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 \\
0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2
\end{pmatrix}$$

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Samuel Boissière, Laboratoire de Mathématiques et Applications, UMR CNRS 6086, Université de Poitiers, Téléport 2, Boulevard Marie et Pierre Curie, F-86962 Futuroscope Chasseneuil
E-mail address: samuel.boissiere@math.univ-poitiers.fr
URL: http://www-math.sp2mi.univ-poitiers.fr/∼sboissie/

Chiara Camere, Universitá degli studi di Milano, Dipartimento di Matematica, via Cesare Saldini 50 20133 Milano, Italy
E-mail address: chiara.camere@unimi.it
URL: http://www.mat.unimi.it/users/camere/fr/index.html

Giovanni Mongardi, Universitá degli studi di Milano, Dipartimento di Matematica, via Cesare Saldini 50 20133 Milano, Italy
E-mail address: giovanni.mongardi@unimi.it

Alessandra Sarti, Laboratoire de Mathématiques et Applications, UMR CNRS 6086, Université de Poitiers, Téléport 2, Boulevard Marie et Pierre Curie, F-86962 Futuroscope Chasseneuil
E-mail address: alessandra.sarti@math.univ-poitiers.fr
URL: http://www-math.sp2mi.univ-poitiers.fr/∼sarti/