Sur les martingales d'Azéma

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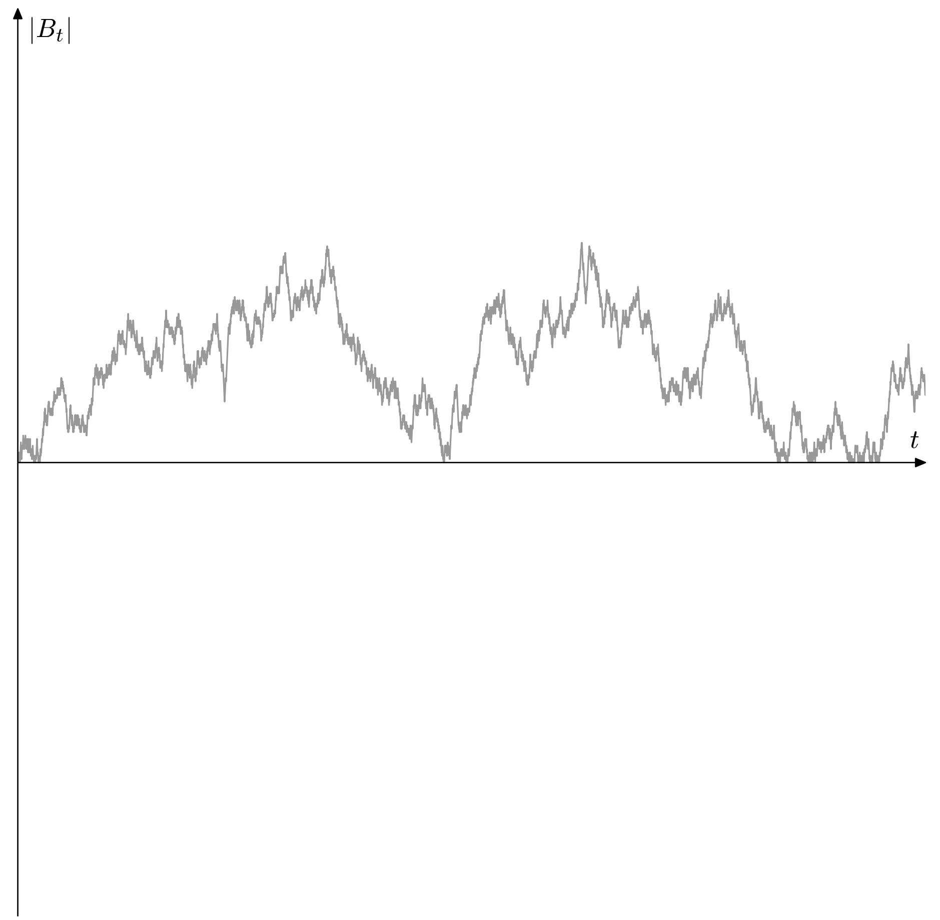
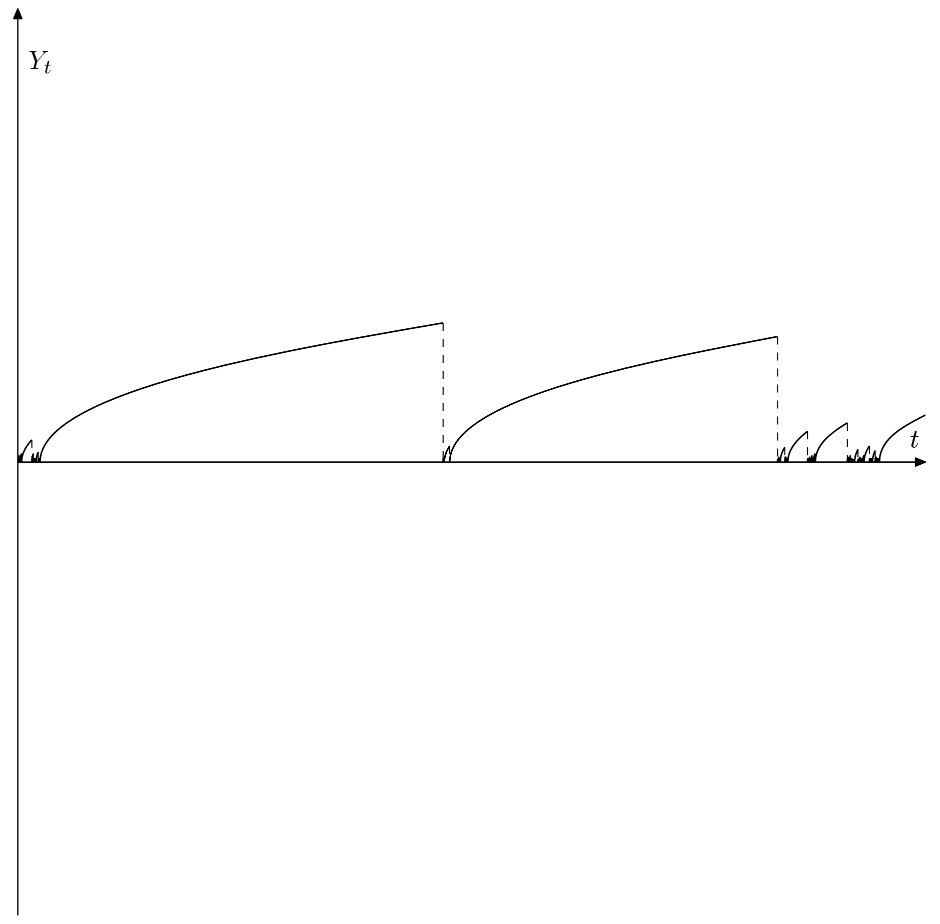
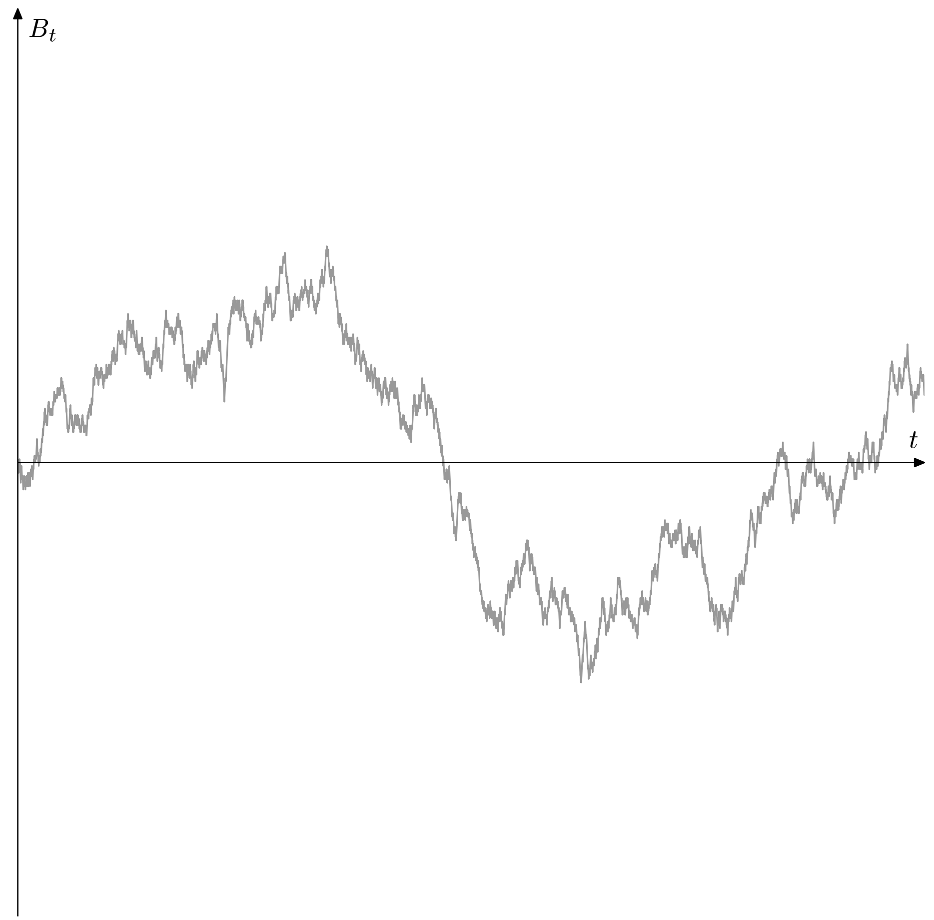
Luminy, 8 septembre 2004

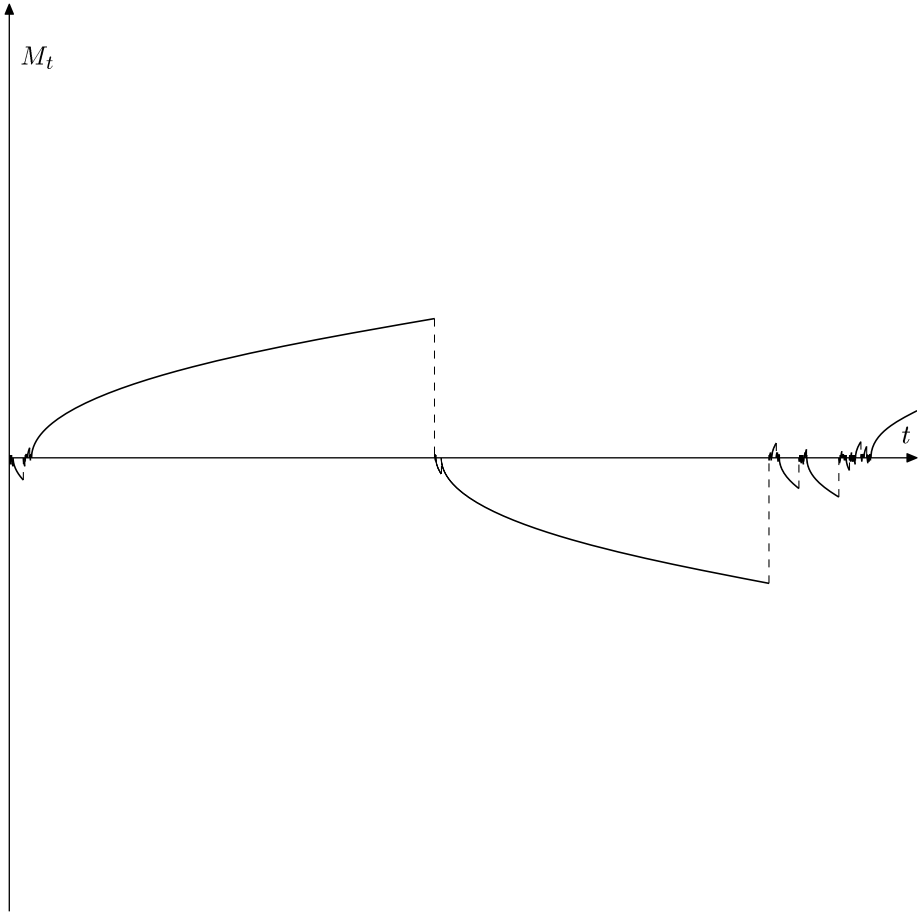
1. Une martingale remarquable

- *Azéma* (J.), « Sur les fermés aléatoires », *Séminaire de probabilités XIX*, Lecture Notes in Mathematics 1123, Springer (1985), p. 397–495.
- *Azéma* (J.) et *Yor* (M.), « Étude d'une martingale remarquable », *Séminaire de probabilités XXIII*, Lecture Notes in Mathematics 1372, Springer (1989), p. 88–130.


$|B_t|$
t


Y_t
t

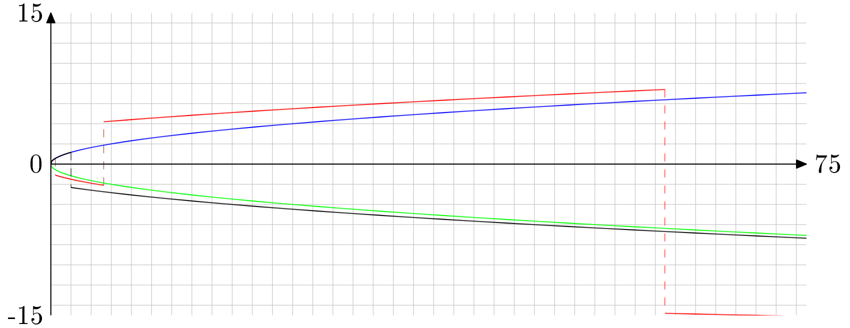

B_t
t

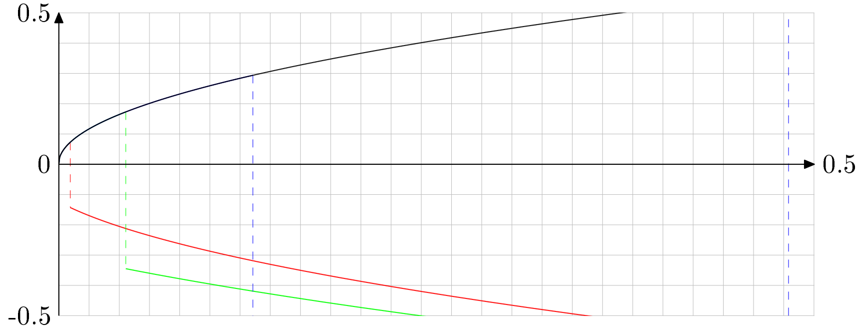

M_t
t

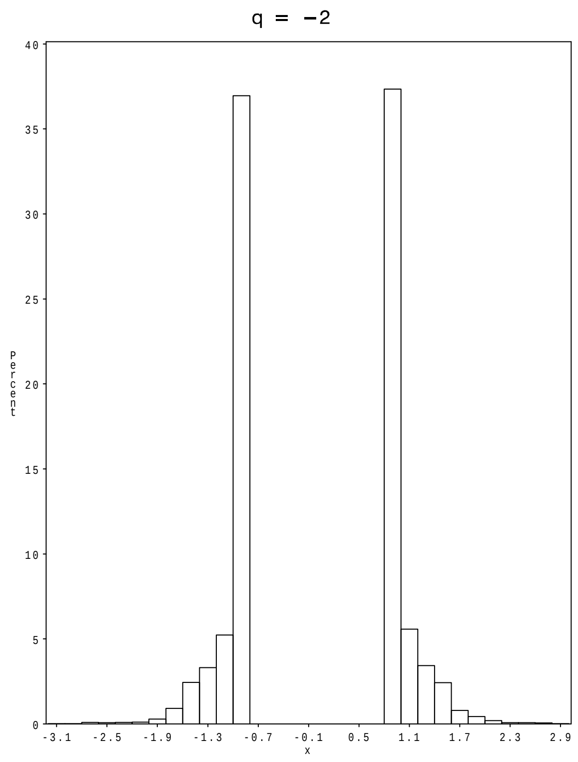
2. **Des martingales remarquables**

- *Émery* (M.), « On the Azéma Martingales », *Séminaire de probabilités XXIII*, Lecture Notes in Mathematics 1372, Springer (1989), p. 66–87.
- ________, « Sur les martingales d'Azéma (suite) », *Séminaire de probabilités XXIV*, Lecture Notes in Mathematics 1426, Springer (1990), p. 442–447.

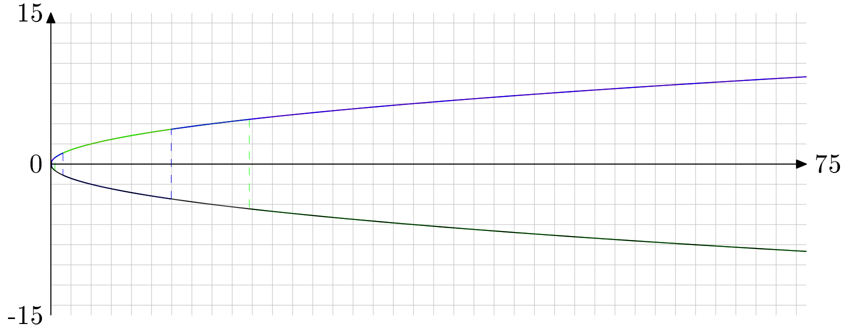
2.1. *Martingales d'Azéma de paramètre* $q = -2$

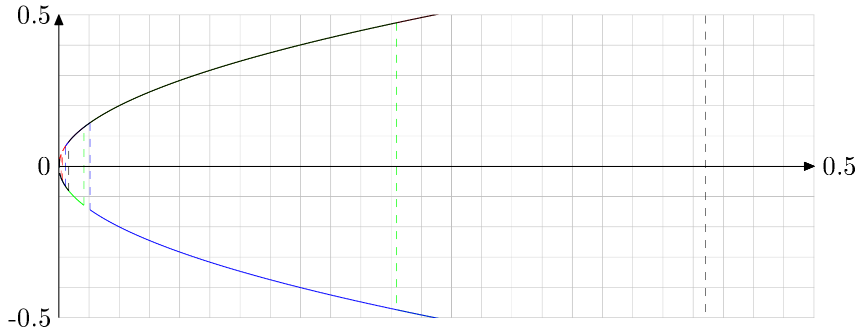


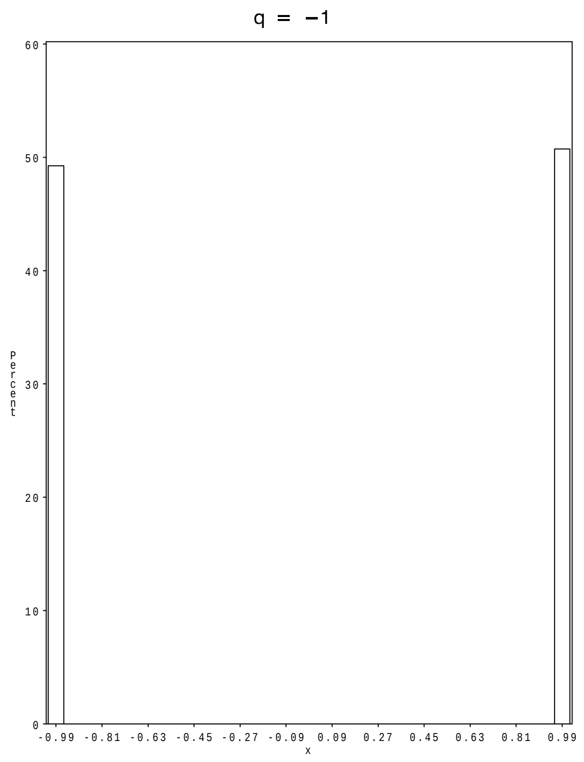




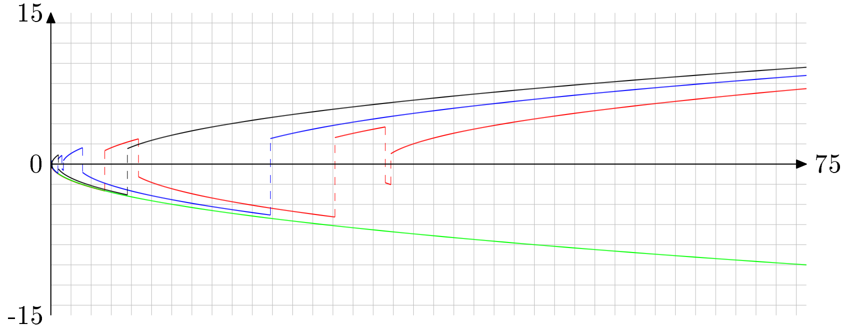
2.2. *Martingales d'Azéma de paramètre* $q = -1$ *(martingale parabolique)*

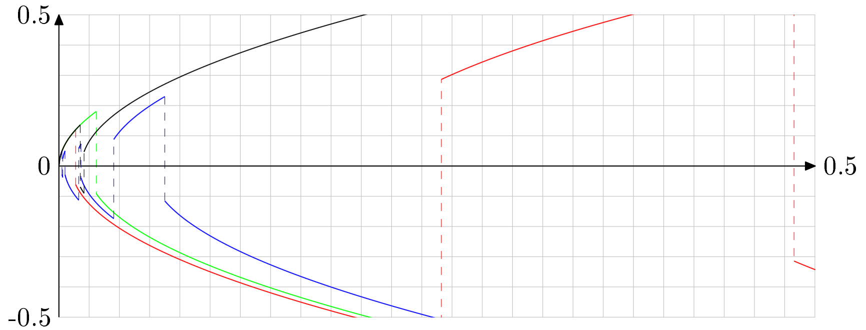


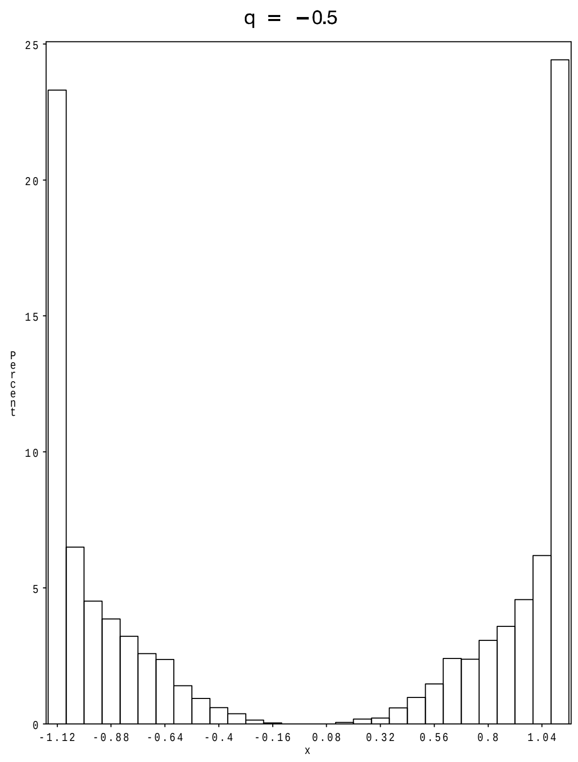




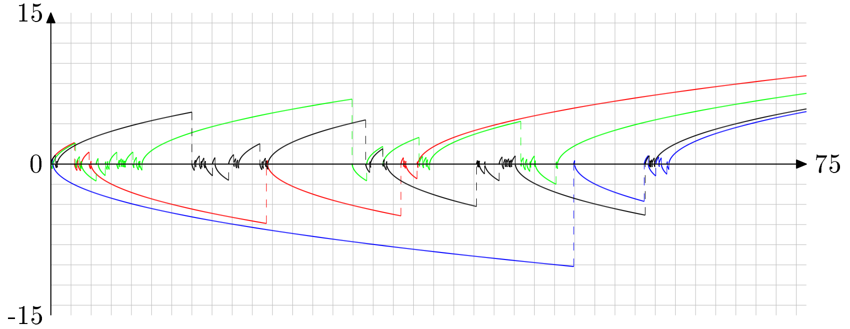
2.3. *Martingales d'Azéma de paramètre* $q = -0{,}5$

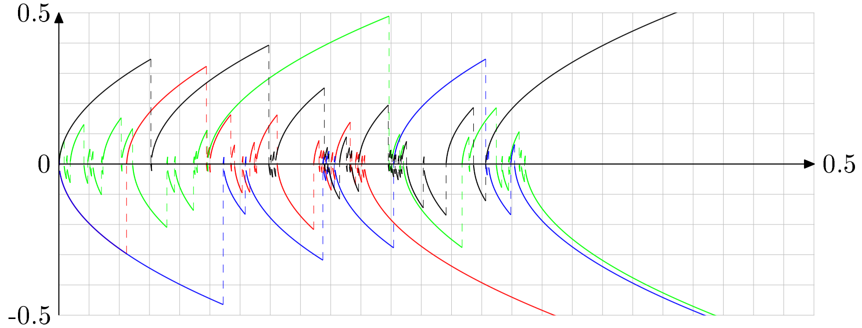


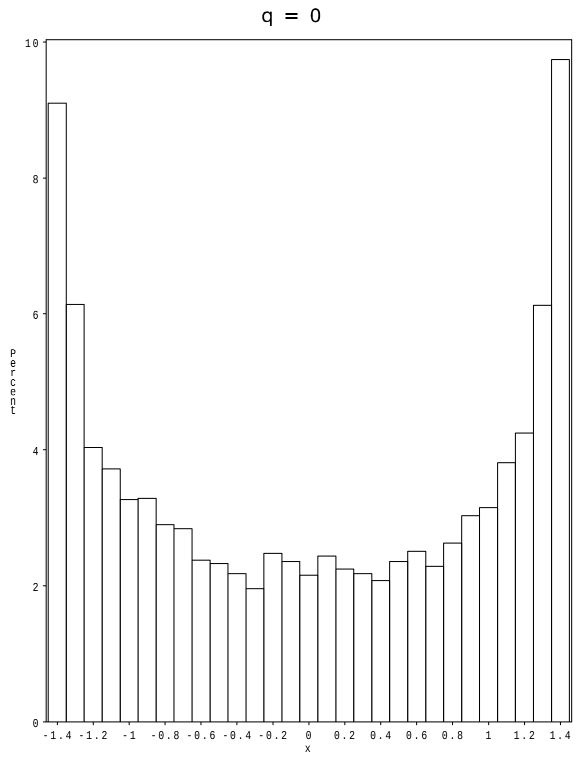




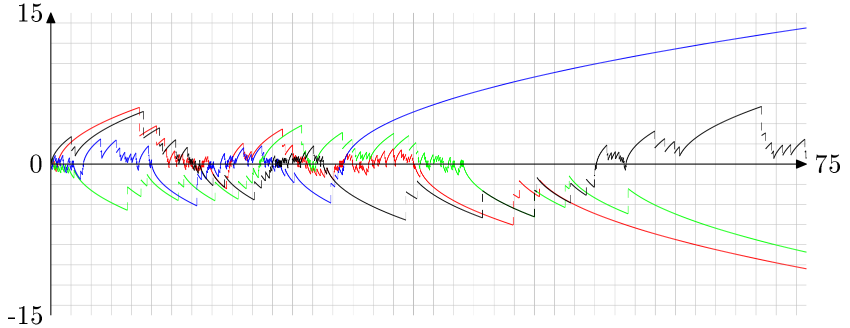
2.4. *Martingales d'Azéma de paramètre* $q = 0$ *(la martingale d'Azéma)*

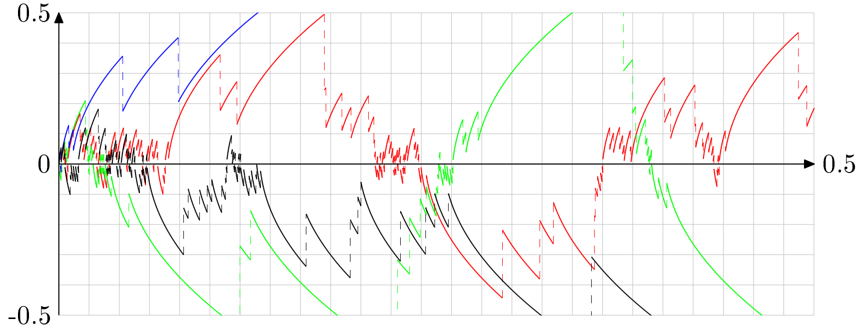


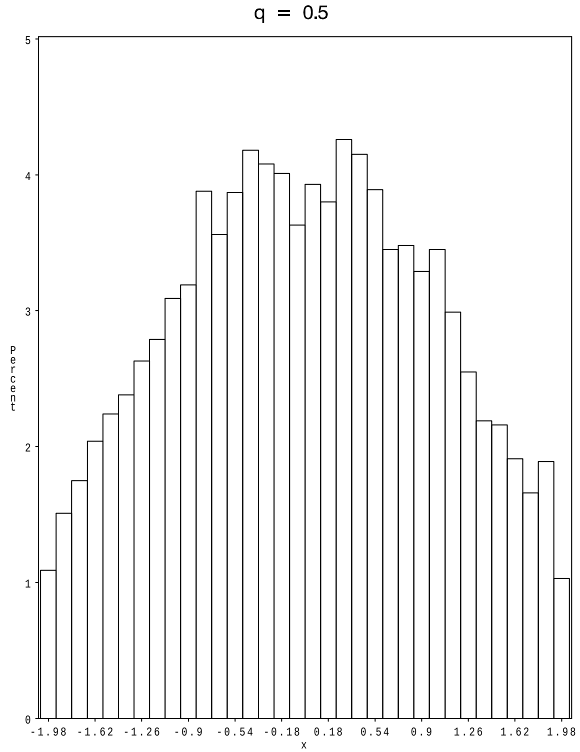




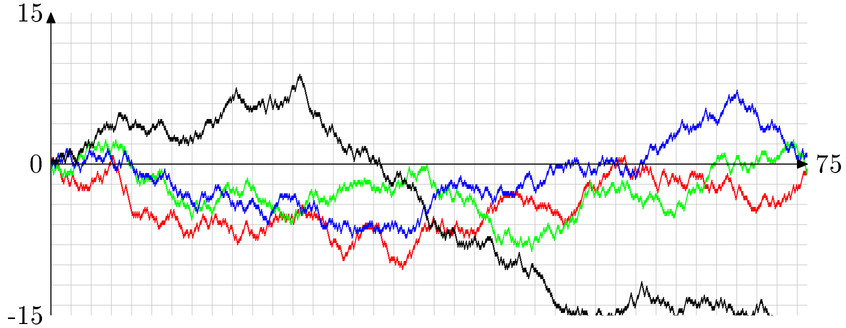
2.5. *Martingales d'Azéma de paramètre* $q = 0{,}5$

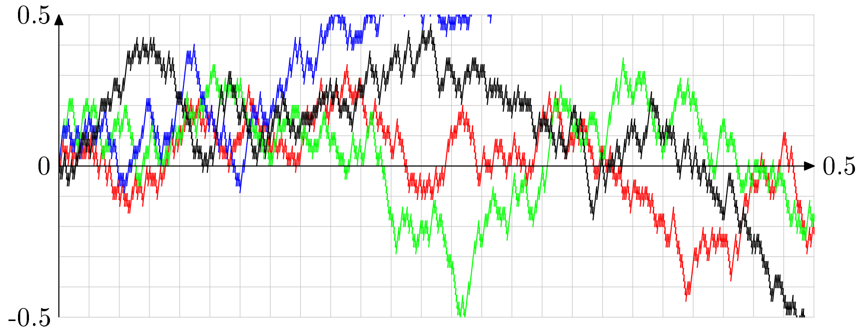


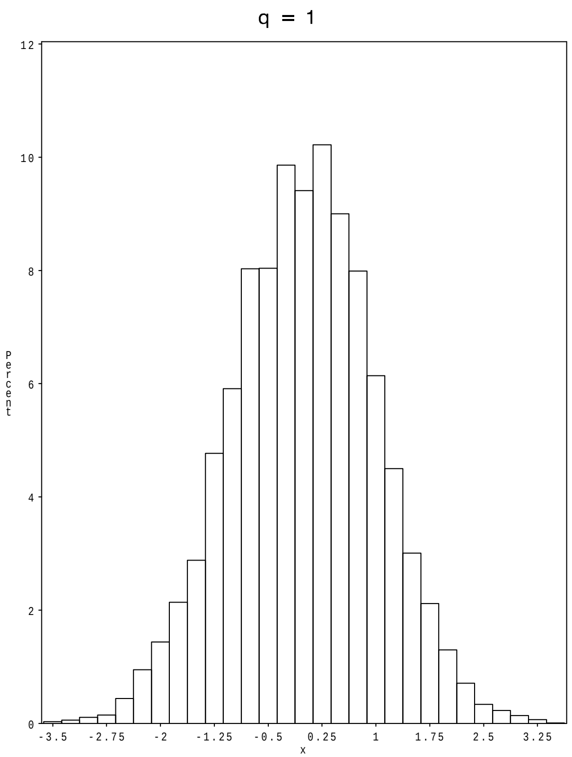




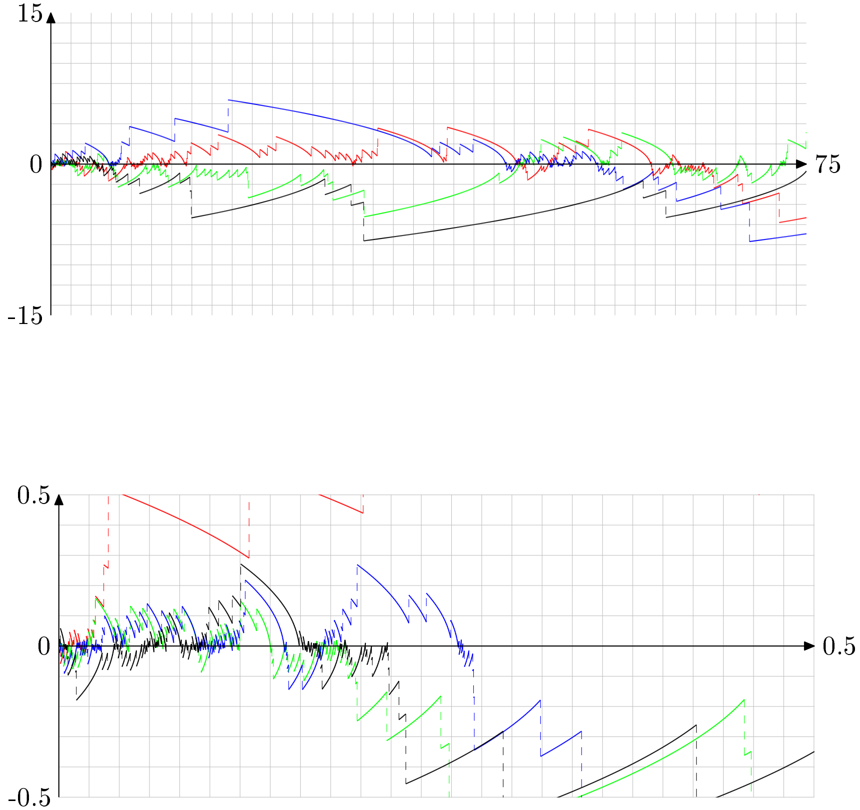
2.6. *Martingales d'Azéma de paramètre $q = 1$ (processus de Wiener)*

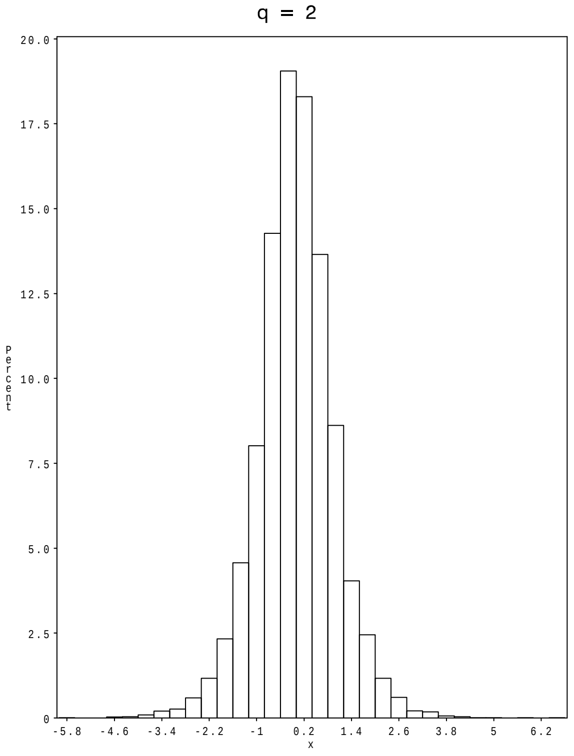






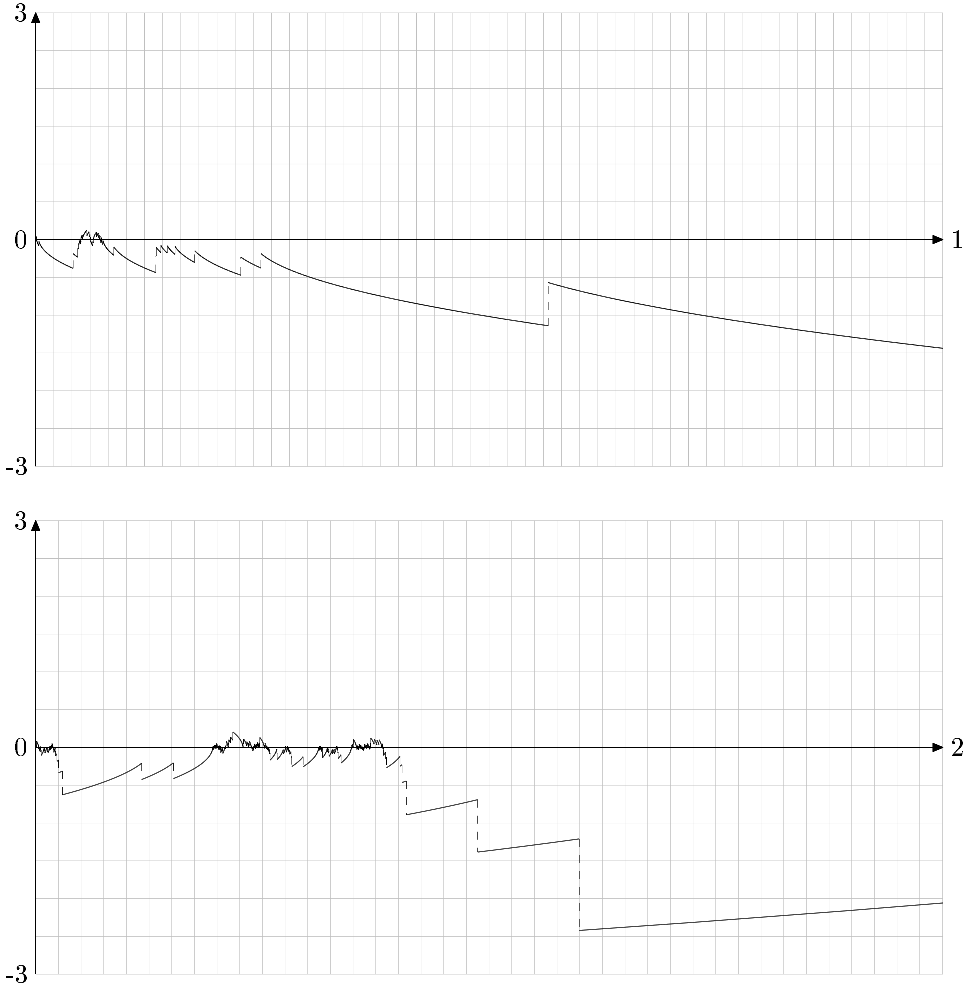
2.7. *Martingales d'Azéma de paramètre* $q = 2$



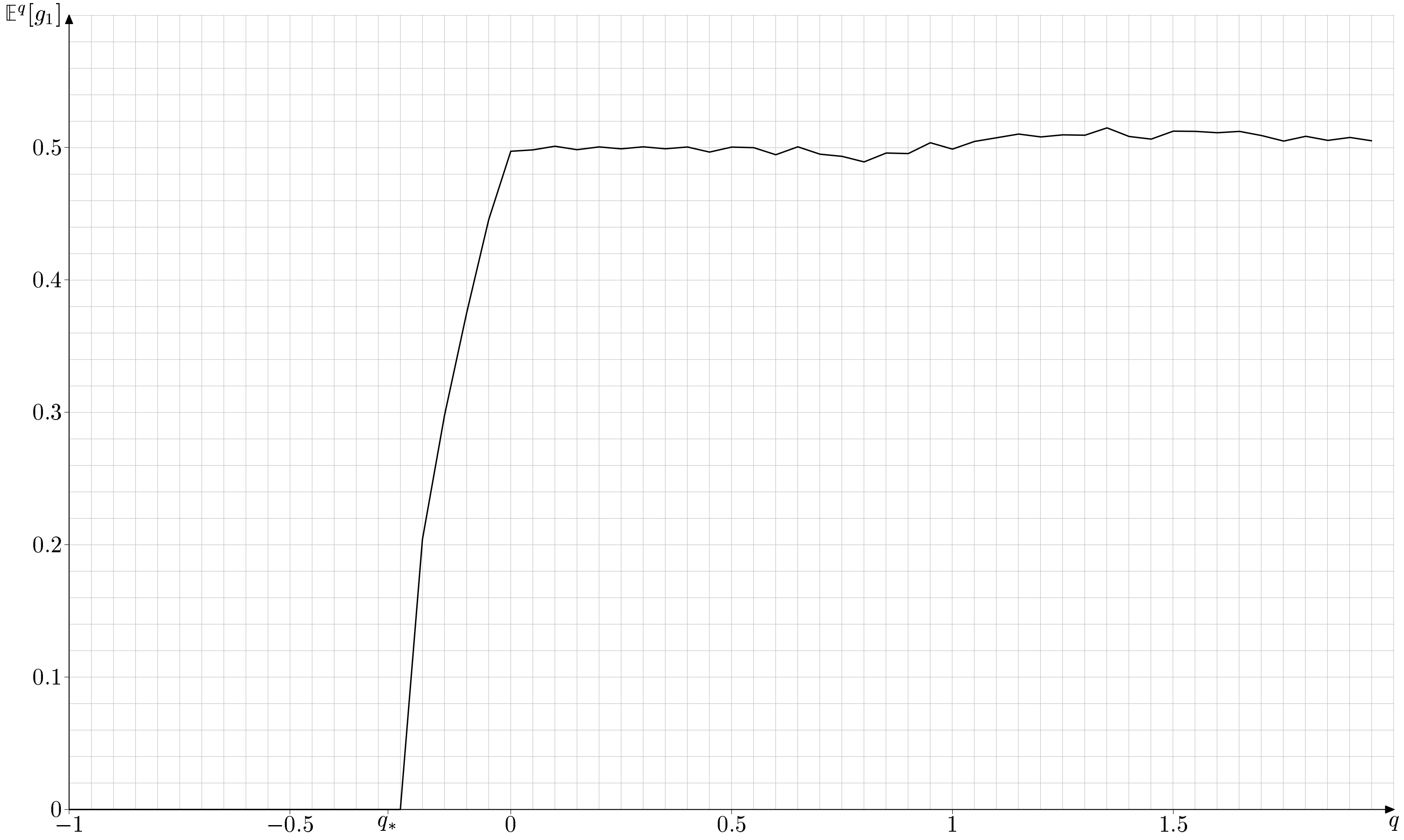


3. **Quelques propriétés**

- Temps locaux
- Propriété de représentation chaotique
- Retournement du temps
- Projection sur la filtration du signe (?)



3
0
1
-3
3
0
2
-3


$\mathbb{E}^q[g_1]$
0.5
0.4
0.3
0.2
0.1
0
−1
−0.5
q_*
0
0.5
1
1.5
q

4. Quelques compléments bibliographiques

- *Azéma* (J.) et *Rainer* (C.), « Sur l'équation de structure, $\mathrm{d}[X, X]_t = \mathrm{d}t - X_{t-}^{+}\, \mathrm{d}X_t$ », *Séminaire de probabilités XXVIII*, Lecture Notes in Mathematics 1583, Springer (1994), p. 236–255.
- *Chao* (T.-M.) & *Chou* (C.-S.), « Some remarks on the martingales satisfying the structure equation $[X, X]_t = t + \int_0^t \beta X_{s-} dX_s$ », *Séminaire de Probabilités XXXV*, Lecture Notes in Mathematics 1755, Springer (2001), p. 87–97.
- *Kurtz* (D.), « Représentation nucléaire des martingales d'Azéma », *Séminaire de Probabilités XXXVI*, Lecture Notes in Mathematics 1801, Springer (2002), p. 457–476.
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- ________, « Azéma martingales with drift », *Probability and Mathematical Statistics*, Vol. 15 (1995), p. 461–468.
- *Protter* (P.), *Stochastic Integration and Differential Equations*, Springer (1990).
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- *Yor* (M.), *Some Aspects of Brownian Motion. Part II: Some Recent Martingale Problems*, Lectures in Mathematics, ETH Zürich, Birkhäuser (1997).