Erratum to "Derivatives and asymptotics of Whittaker functions"

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Abstract

We correct the statement of Theorem 2.1 of [M], which makes no sense as it is written. We simply forgot to take in account the contribution of the zero derivatives to the asymptotic expansion of Whittaker functions on the torus. The mistake is minor as they in fact don't really contribute, or more precisely they do via Schwartz functions which vanish at zero. The proof remains unchanged except for some details that we explain here.

1 The correct expansion and its proof

The arxiv version http://arxiv.org/abs/1004.1315 of the paper has been directly modified, and the statement and its proof there are correct.

We use the notations of [M], and consider π a θ-generic representation of G_n. In particular, natural embeddings of G_i inside G_n are defined in [ibid.], as well as central subgroups Z_i of G_i, so that the group $A_n = Z_1 \dots Z_n$ is a maximal torus of G_n. We consider the set R of positive integers between 1 and $n-1$ such that the nonzero derivatives of π with respect to θ are the finite length representations $\pi^{(n-k)}$ of G_k, for $k \in R$. We then denote for each $k \in R$, by $(c_{l_k,k})_{l_k}$ the family of characters of Z_k given by the action of this group on the irreducible sub-quotients of $\pi^{(n-k)}$. We finally set $V = \{1, \dots, n-1\} - R$. The correct statement of Theorem 2.1 is the following.

Theorem 1.1. *For any W in $W(\pi,\theta)$, the function*

$$W(z_1 \dots z_{n-1})$$

is a linear combination of products of the form

$$\prod_{k\in R} [c_{l_k,k}\delta_{U_k+1}^{1/2} \dots \delta_{U_n}^{1/2}](z_k) v^{m_k}(z_k)\phi_k(z_k) \prod_{l\in V} \phi_l(z_l),$$

for non negative integers m_k, functions ϕ_k in $\mathcal{C}_c^\infty(Lie(Z_k))$, and functions ϕ_l in $\mathcal{C}_c^\infty(Lie(Z_l))$ vanishing at $z_l = 0$.

We now explain how the proof in [M] should be modified to obtain the above statement.

Proof. First, in the proof of Theorem 2.1 in [M], what is proved is in fact a stronger statement (see the beginning of the proof where this statement is explicitely stated), which implies the one above. This stronger statement should be modified exactly as above as well.

Then the special case $n = 2$ is treated as an example. In this case, if $\pi^{(1)}$ is nonzero, the proof in [M] is correct, whereas if not, the proof is much simpler. Indeed, in this case, the function $W(z_1)$ vanishes for z_1 near zero according to Proposition 2.3 of [M] and also vanishes for $|z_1|$ large according to Remark 3.1, i.e. with the notations of [M], one can take $S = 0$ and $W = D$, and this ends the proof in this case.

Finally, the general case is by induction on n. In this case, if $\pi^{(1)}$ is nonzero, the proof in [M] is correct, except that one replaces the induction hypothesis statement by the correct one. If $\pi^{(1)}$ is equal to zero, then the proof is simpler again. Indeed one can again take $S = 0$ and $W = D$ directly, and the rest of the proof in [M] remains unchanged, except that one replaces the induction hypothesis statement by the correct one. This ends the general proof. □

In Theorem 3.1 and Corollary 3.1, the characters $c_{i_k,n-k}$ for k between 1 and $n-1$ should also be changed to the characters $c_{i_k,k}$ for $k \in R$.

References

[M] N. Matringe, *Derivatives and asymptotics of Whittaker functions*, Represent. Theory, 13 (2011), Volume 15, Pages 646-669