CORRIGENDUM AND IMPROVEMENT FOR "CHAOTIC SOLUTION FOR THE BLACK-SCHOLES EQUATION"

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Abstract. *We correct an error and improve the main result in our paper: Chaotic solution for the Black-Scholes equation, Proceedings of the American Mathematical Society 140 (2012), 2043–2052.*

In the proof of Lemma 3.5, the function we denoted by $g(z) = \nu^2 z^2 + (r - \nu^2)z - r$ should have been according to (3.2), $z^2 + (r/\nu - \nu)z - r$. Thus

$$\operatorname{Re} g(z) = x^2 - y_0^2 + (\frac{r}{\nu} - \nu)x - r = 0$$

with $z = x + \mathrm{i}y_0$. We must find (x, y_0) with $0 < x < \nu s, y_0 \in \mathbb{R}$ such that

$$x^2 + (\frac{r}{\nu} - \nu)x - r = y_0^2. \tag{3.3}$$

Call $\mathscr{C}$ the curve represented by the graph of the quadratic function $y = x^2 + (\frac{r}{\nu} - \nu)x - r$. As Figure 1' shows, for $\nu < x < \nu s$, there are uncountably many points (x, y) on the dashed portion of $\mathscr{C}$ with $y > 0$. For each such point let $y_0 = \sqrt{y}$. Then this gives uncountably many solutions of (3.3).

With this correction in the proof, our main results, Theorems 3.6 and 3.7, have the same conclusions under weaker hypotheses. The following is a precise statement of this.

THEOREM 3.6'. Let $s > 1$, $\tau \geq 0$ and define the complex Banach space

$$Y^{s,\tau} := \{u \in C(0,\infty) : \lim_{x\to 0} \left|u(x)/(1 + x^{-\tau})\right| = \lim_{x\to 0} |u(x)/(1 + x^s)| = 0\}$$

with norm

$$\|u\|_{s,\tau} = \sup_{x>0} \left|u(x)/[(1 + x^{-\tau})(1 + x^s)]\right|.$$

The Black-Scholes equation

$$\partial v/\partial t = (\sigma^2/2)x^2\partial^2 v/\partial x^2 + rx\partial v/\partial x - rv,$$

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for $\sigma > 0$, $r > 0$, is governed by a (C_0) semigroup $T = \{T(t) : t \geq 0\}$ on $Y^{s,\tau}$.This semigroup is chaotic. If $Y^{s,\tau}_{\mathbb{R}}$ consists of the real functions in $Y^{s,\tau}$, then S_T, the restriction of T to $Y^{s,\tau}_{\mathbb{R}}$, is a chaotic (C_0) semigroup.

Thus we do not need the assumption $\sigma s > \sqrt{2}$,which was made in the original Theorems 3.6, 3.7. Consequently the choice of spaces for the chaosity of the Black-Scholes semigroup is independent of the volatility σ, which gives a conceptually cleaner and improved result.

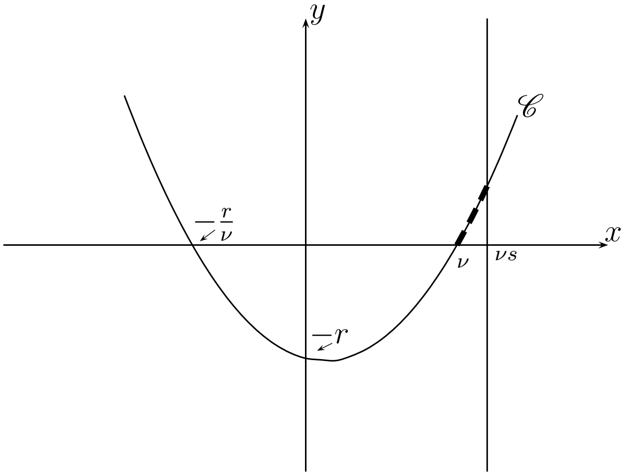

Figure 1'.

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