# The sheets of a Lie algebra and the one dimensional representations of finite W-algebra

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Let $G$ be reductive over $\mathbb{C}$ and let $\mathfrak{g} = \mathrm{Lie}(G)$. For $e \in \mathfrak{g}$ nilpotent we study the finite W-algebra $U(\mathfrak{g}, e)$. The one dimensional $U(\mathfrak{g}, e)$-modules are parameterised by the maximal spectrum of the maximal abelian quotient $E(\mathfrak{g}, e) := \mathrm{Specm}(U(\mathfrak{g}, e)^{\mathrm{ab}})$. We show that $U(\mathfrak{g}, e)^{\mathrm{ab}}$ is generated by $\dim \mathfrak{g}_e^{\mathrm{ab}}$ variables where $\mathfrak{g}_e^{\mathrm{ab}} := \mathfrak{g}_e/[\mathfrak{g}_e, \mathfrak{g}_e]$. In type A, $E(\mathfrak{g}, e)$ is known to be isomorphic to an affine space of dimension $\dim \mathfrak{g}_e^{\mathrm{ab}}$. I shall classify the nilpotent elements in other classical types such that the same holds. It turns out that this is if and only if $e$ lies in a unique sheet of $\mathfrak{g}$ and I'll give a complete combinatorial description of such orbits in terms of partitions. Furthermore I shall describe the geometry of the fixed points of $E(\mathfrak{g}, e)$ under the natural action of the component group (this has an even nicer picture).

I will give some applications to classical problems regarding completely prime primitive ideals of $U(\mathfrak{g})$ and quantisations of the orbit $\mathrm{Ad}(G)e$.