★★★ Journée Hassan Emamirad ★★★

★ ★ ★ Poitiers 9 juin 2011 ★ ★ ★

Université de Poitiers
Laboratoire de Mathématiques et Applications
Bd. Pierre et Marie Curie, Téléport 2
86962 Futuroscope Chasseneuil Cedex, France

Conférenciers invités

Mickaël Balabane – Claude Bardos – Jerome A. Goldstein
François Golse – Khalid Latrach – Mustapha Mokhtar-Kharroubi

Organisateurs : Saïd Hilout – Philippe Rogeon

Les organisateurs remercient Jocelyne Attab, Brigitte Brault, Benoît Métrot, Nathalie Mongin, Ali Al Riyabi, Houssam Chrayteh et Haydi Israel pour leur aide

Programme

À partir de 9h15 : Accueil

10h15 – 11h : C. Bardos : ***Du théorème de Krein–Rutman au modèle FENE (Finite Extension in Nonlinear Elasticity)*** – Chairman : J.A. Goldstein

11h05 – 11h50 : M. Mokhtar–Kharroubi : ***Perturbation theory for Schrödinger–type Hamiltonians*** – Chairman : J.A. Goldstein

11h50 – 14h : ***Déjeuner***

14h15 – 15h : F. Golse : ***Homogenization of the linear Boltzmann equation in a periodic system of holes*** – Chairwoman : G. Goldstein

15h05 – 15h50 : K. Latrach : ***Existence results for a nonlinear transport equations in bounded geometry on L^1–spaces*** – Chairwoman : G. Goldstein

15h50 – 16h10 : ***Pause Café***

16h15 – 17h : M. Balabane : ***Functional analysis for Helmholtz equation in the framework of domain decomposition*** – Chairman : J.L. Bona

17h05 – 17h50 : J.A. Goldstein : ***The Black–Scholes semigroup of mathematical finance is chaotic*** – Chairman : J.L. Bona

Participants

1. **H. Emamirad, Poitiers**
2. **M. Balabane, Paris 13**
3. **C. Bardos, Paris 6**
4. **J.A. Goldstein, Memphis, USA**
5. **F. Golse, Ecole Polytechnique**
6. **K. Latrach, Clermont–Ferrand II**
7. **M. Mokhtar–Kharroubi, Besançon**
8. **A. Miranville, Poitiers**
9. **R. Aftabizadeh, Ohio**
10. **G. Goldstein, Memphis, USA**

11. **J.L. Bona, Illinois à Chicago**
12. **H. Chen, Memphis, USA**
13. **F. Moukamba, Brazzaville**
14. **M. Grasselli, Polotecnico Milan**
15. **Y. Dermenjian, Aix–Marseille I**
16. **P. Rogeon, Poitiers**
17. **I. Laadnani, Poitiers**
18. **S. Hilout, Poitiers**
19. **T. El Arwadi, Marseille**
20. **H. Chrayteh, Poitiers**
21. **H. Israel, Poitiers**
22. **B. Doumbé Bangola, Poitiers**
23. **J. Grilhé, Poitiers**
24. **A. Al Riyabi, Poitiers**
25. **J.M. Rakotoson, Poitiers**
26. **A. Abourou Ella, Poitiers**
27. **G. Sadaka, Poitiers**
28. **A. Cimetiere, Poitiers**
29. **B. Saoud, Poitiers**
30. **M. Pierre, Poitiers**
31. **A. Rougirel, Poitiers**
32. **M.A. Cherif, Sfax, Tunisie**
33. **E. Minto'o Azariel Paul, de Poitiers**
34. **M. Boutat, Poitiers**
35. **C. Vallée, Poitiers**
36. **L. Cherfils, La Rochelle**
37. **I. Mortazavi, Bordeaux**
38. **S. Huberson, Poitiers**
39. **J. Dambrine, Poitiers**

Résumé de l'exposé de M. Balabane

Functional analysis for Helmholtz equation in the framework of domain decomposition.
The aim of the lecture is to describe a Domain Decomposition algorithm for the Helmholtz equation. Given a bounded open set $\Omega = \Omega_1 \cup \Omega_2 \cup \Gamma$ where the two open sets Ω_1 and Ω_2 are not overlapping, and Γ a common subset of their boundary (called the fictitious boundary), and given a (global) solution u of the Helmholtz equation

$$\Delta u + k^2 u = f \in L^2(\Omega) \quad \text{and} \quad u \in H_0^1(\Omega)$$

the aim is to understand the dynamics of the sequence $(v_1^n, v_2^n)_{n\in\mathbb{N}}$ solving separatly the Helmholtz equations on Ω_1 and Ω_2, when equating the fluxes through Γ : ($m = 1, 2$ resp. $m' = 2, 1$)

$$\frac{\partial v_m^n}{\partial n_m} - i\gamma v_m^n = -\frac{\partial v_{m'}^{n-1}}{\partial n_{m'}} - i\gamma v_{m'}^{n-1} \quad \text{on} \quad \Gamma$$

The ultimate aim is to prove convergence to $(u_{|\Omega_1}, u_{|\Omega_2})$ of the sequence $(u_1^n, u_2^n)_{n\in\mathbb{N}}$ solving the Helmholtz equations on Ω_1 and Ω_2 with a relaxation on the boundary Γ added, namely :

$$\frac{\partial u_m^n}{\partial n_m} - i\gamma u_m^n = \theta[\frac{\partial u_m^{n-1}}{\partial n_m} - i\gamma u_{n-1}^2] - (1-\theta)[\frac{\partial u_1^{n-1}}{\partial n_{m'}} + i\gamma u_{m'}^{n-1}] \quad \text{on} \quad \Gamma$$

For this sake, the geometry of the set of solutions of the Helmholtz equation on $\Omega_1 \times \Omega_2$ with equated energy fluxes is studied, through the study of the coupling operator defined on $L^2(\Gamma)\times L^2(\Gamma)$ which intertwins the fluxes. It turns out that the key for understanding the convergence of the sequence $(u_1^n, u_2^n)_{n\in\mathbb{N}}$ is the analysis of the spectral properties of the intertwinnig operator.

Using these tools, one can prove that the Domain Decomposition setting for the Helmholtz equation leads to an ill–posed problem. Nevertheless, one can prove that if a solution exists, it is unique, and that the algorithm do converge to the solution.

Convergence of the relaxed algorithm is proven and numerical tests for solving the Helmholtz equation through this domain decomposition algorithm are given.

Résumé de l'exposé de C. Bardos

Du théorème de Krein–Rutman au modèle FENE (Finite Extension in Nonlinear Elasticity) Pour modéliser les polymères, on couple deux équations : une équation de type Navier–Stokes pour l'évolution macroscopique du fluide et une autre de type Fokker Planck engendrant un semi groupe linéaire pour les variables mircoscopiques décrivant la densité de polymères en interaction avec le fluide. Le modèle FENE s'intéresse à des polymères de taille finie R donc à des variables d'espace dans un disque de rayon R. Le fait qu'il s'agisse de polymères de taille maximum R conduit à l'introduction au niveau microscopique d'un opérateur du second ordre elliptique dégénérant sur la frontière du disque de rayon R, la valeur propre principale et le vecteur propre principal correspondant jouent un rôle essentiel dans le comportement asymptotique (pour $t \longrightarrow \infty$) de l'ensemble. C'est alors qu'il convient d'introduire une version bien adaptée du théorème de Krein–Rutman pour prouver la simplicité de la valeur propre dominante et la positivité du vecteur propre correspondant.

Résumé de l'exposé de J.A. Goldstein

The Black–Scholes semigroup of mathematical finance is chaotic. To nonmathematicians, the three standard everyday examples of "chaos" are the weather, marriage, and the stock market. Weather prediction led to the theory of chaos, which was thought to be solely a nonlinear phenomenon. But in recent years it was discovered that chaos could occur in infinite dimensional linear situations. There are nice "toy" examples of this, but these examples do not arise from a scientific context. Of the three examples mentioned above, the first two are clearly nonlinear. But in developing a theory of stock options in the 1970s, Black, Merton and Scholes derived an important linear parabolic PDE that greatly advanced mathematical finance and earned a Nobel Prize in Economics. As was typical in a Markov process context, the governing semigroup was contractive on a sup norm space, and hence nonchaotic as it could not have a dense orbit. But the correct "initial value" (from their derivation) for their stock option equation was an unbounded function. So the "right" space or spaces for the problem had to be found. We found candidates for these spaces and proved that the corresponding BMS semigroup is chaotic on these spaces. The spaces are

$$Y^{s,\tau} = \{u \in C_0(0,\infty) : \lim_{x\to\infty} \frac{u(x)}{1+x^s} = 0, \quad \lim_{x\to 0} \frac{u(x)}{1+x^{-\tau}} = 0\},$$

with norm $\|u\|_{Y^{s,\tau}} = \sup\limits_{x>0} \left| \frac{u(x)}{(1+x^s)(1+x^{-\tau})} \right| < \infty$. Our main theorem is that the Black-Scholes semigroup is strongly continuous and chaotic on $Y^{s,\tau}$for $s > 1$, $\tau \geqslant 0$, with $s\sigma > \sqrt{2}$, where σ is the volatility. The proof uses the Godefroy-Shapiro hypercyclicity criterion. The lecture will attempt to explain all this and more.

Résumé de l'exposé de F. Golse

Homogenization of the linear Boltzmann equation in a periodic system of holes. This talk reviews recent results obtained in collaboration with E. Bernard and E. Caglioti on the homogenization problem for the linear Boltzmann equation for a monokinetic population of particles set in a periodically perforated domain, assuming that particles are absorbed by the holes. We distinguish a critical size for the hole radius in terms of the distance between neighboring holes, derive the homogenized equation under this scaling assumption, and study the asymptotic mass loss rate in the long time limit. The homogenized equation so obtained is set on an extended phase space as it involves an extra time variable, which is the time since the last jump in the stochastic process driving the linear Boltzmann equation. The asymptotic decay of the mass follows from elementary arguments in renewal theory.

Résumé de l'exposé de K. Latrach

Existence results for a nonlinear transport equations in bounded geometry on L^1–spaces. In this talk we present some existence results for a multidimensional nonlinear transport equation in bounded geometry on L^1–sapces. The problem may be transformed into a fixed point problem of the form

$$A\psi + B\psi = \psi. \tag{$\bigstar$}$$

But, due to the lack of compactness of the involved operators on L^1–spaces, the classical fixed point theorem of Krasnoselskii does not work. Using recent versions of the Darbo and Krasnoselskii fixed point theorems for the weak topology we show that, for each $r > 0$, the problem ($\bigstar$) has at least one solution on B_r where B_r denotes the closed ball centered at zero with radius r. The main arguments in our analysis are the dissipativity of the streaming operator (for dissipative boundary conditions) and the fact that one of the operators above (for example A) maps weakly compact sets of L^1 into norm compact ones.

Résumé de l'exposé de M. Mokhtar–Kharroubi

Perturbation theory for Schrödinger–type Hamiltonians. We present a new functional analytic approach of Schrödinger Hamiltonians relying on L^1 tools (in particular on local weak-compactness arguments) which extends the Kato class potentials.This formalism has a much more larger scope and applies for instance to generators of convolution semigroups covering thus generators of α-stable semigroups, relativistic Schrödinger semigroups etc. We show also how to derive form-bounds for "multi-particles" Schrödinger-type Hamiltonians.

★★★ Merci de votre participation ★★★