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TOPOLOGICAL PHASE TRANSITIONS

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Phase transitions are phenomena occurring in large random systems, as water cools and turns to ice, is heated and turns to vapor, or as iron loses or gains magnetism. Although not usually recognised as such, there is topology in these transitions, generally studied only at the topologically simple level of connectivity.

Recently, there has been growing interest in far more sophisticated phase transitions, studying large random topological structures undergoing transitions in their homological structure. The results in this area, to a large extent initially motivated by questions from topological data analysis, are building the foundations of a new area of research, that one might call Random Topology.

The aim of the lecture will be to describe some of these developments, mainly via a number of examples, ranging from random topological noise known as crackle to a version of a random Morse theory. While the details of the underlying mathematics are not simple, the examples are easy to follow and provide an instructive path into random topology.

STOCHASTIC EULER-POINCARÉ REDUCTION

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We prove a Euler-Poincaré reduction theorem for stochastic processes taking values in a Lie group. This consists in finding the equation satisfied by drifts of processes which are extremal for some energy functional. We show examples of its application to SO(3) and to the group of diffeomorphisms, which includes the Navier-Stokes equation on a bounded domain and the Camassa-Holm equation.

STATISTICAL MODELS FOR FACE IMAGE ANALYSIS

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The analysis and understanding of visual objects such as faces from video yields to the notion of "non-rigid" textured objects. While sharing a common semantic denominator, face images are expressed by an infinite variety of numbers resulting from the many shapes and textures that identity, expression, illumination or pose changes can generate. In this context of high-dimensional, high-variation data description, statistical analysis is an almost mandatory step to define a set of key numerical elements that will be discrimininative enough for identity, age, gender or emotion recognition. Using tools such as Principal Component Analysis (PCA), we are able to produce a dual analysis/synthesis modeling scheme similar to other decomposition techniques well-known within the signal processing community (wavelets, polynomial decomposition, etc.). Based on a summerized description of examples and counterexamples, machine learning algorithms are then able to achieve the difficult task of face recognition and classification.

CONFORMAL MEASURE ENSEMBLES

Federico Camia
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Several classical lattice models of statistical mechanics, such as percolation and the Ising and Potts models, can be described in terms of clusters. In the last fifteen years, there has been tremendous progress in the study of the geometric properties of such models in two dimensions in the scaling limit, when the lattice spacing is sent to zero. Much of that work has focused on cluster boundaries, using the Schramm-Loewner Evolution (SLE), introduced by Schramm, and the Conformal Loop Ensembles (CLEs), introduced by Sheffield and Werner. In this talk I will be concerned with the scaling limit of the clusters themselves and their "areas." This leads to the study of rescaled counting measures and to the concept of Conformal Measure Ensembles, with interesting applications to two-dimensional critical percolation and the two-dimensional critical Ising model. (Based on joint work with R. Conijn and D. Kiss.)

TBA

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ORDER STATISTICS FOR THE INRADIUS OF A POISSON LINE TESSELLATION

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A Poisson line tessellation is observed in the window $\mathbf{W}_\rho = B(0, \pi^{-1/2}\rho^{1/2})$, for $\rho > 0$. With each cell of the tessellation, we associate the inradius, which is the radius of the largest ball contained in the cell. Using Poisson approximation, we compute the limit distribution of the largest and smallest order statistics for the inradii of all cells whose incenters are contained in $\mathbf{W}_\rho$ as ρ goes to infinity. We additionally prove that the limit shape of the cells minimizing the inradius is, with high probability, a triangle.

SUBLINEARITY OF THE MEAN NUMBER OF SEMI-INFINITE BRANCHES FOR GEOMETRIC RANDOM TREES

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Since the seminal work of Howard and Newman (in 2001), the straight character of many geometric random trees has been put forward. In particular, these trees admit semi-infinite branches in all the directions. In this talk, we develop a general method ensuring that the mean number of these semi-infinite branches is sublinear. Our method essentially relies on a local approximation (in distribution) of the tree far away from its root by a suitable directed forest and on the fact that this directed forest a.s. admits only one topological end. We apply this method to two geometric random trees of very different nature.

THE POISSON-VORONOI CELL AROUND AN ISOLATED NUCLEUS

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Consider a random point process made of the union of a point (the origin) with a Poisson Point process having a (high) uniform intensity outside a deterministic set surrounding the origin. It is then pretty obvious that, when the intensity goes to infinity, the Voronoi cell associated to the origin converges from above to a deterministic convex set. We will describe this set and give the asymptotics of the exceeding area, exceeding perimeter and number of vertices of the Voronoi cell. In a way, these results can be considered as dual with respect to the results of Rényi and Sulanke who considered through random convex hulls random approximations of convex sets from below. It is interesting to see how old results about pedal curves play a role in this problem. This is a joint work with Pierre Calka and Yann Demichel.

TRAFFIC IN A RANDOM POISSONIAN NETWORK: LIMIT THEOREMS ASSISTED BY IMPROPER POISSON LINE PROCESSES.

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A random spatial network: in a large disk can be constructed using a stationary and isotropic Poisson line process of unit intensity, with a suitable "cross-country" convention to connect points not lying on the network [1]. Suppose traffic is generated uniformly along "near-geodesics" (connecting pairs of points using a certain lazy algorithm). If the Poisson line pattern is conditioned to contain a line through the centre, then one can show that a scaled version of the traffic flow has asymptotic distribution given by the 4-volume of a region in 4-space, constructed using an improper anisotropic Poisson line process in an infinite planar strip. (Improper Poisson line processes are pervasive in this area: see also the "Poisson line SIRSN" [3,4]). A more amenable representation can be obtained in terms of two seminal curves defined by the improper Poisson line process, thus producing a framework for effective simulation from this distribution up to an L^1 error which tends to zero with increasing computational effort [2].

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TBA.

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SCALING LIMIT OF DYNAMICAL PERCOLATION ON ERDÖS-RÉNYI CRITICAL RANDOM GRAPHS.

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Consider a critical Erdös-Rényi random graph: n is the number of vertices, each one of the $\binom{n}{2}$ possible edges is kept in the graph independently from the others with probability $p(n) = n^{-1} + \lambda n^{-4/3}$, λ being a fixed real number. When n goes to infinity, Addario-Berry, Broutin and Goldschmidt have shown that the collection of connected components, viewed as suitably normalized compact connex metric measure spaces, converge in distribution to a continuous limit made of random real graphs closely linked to the brownian random tree of Aldous. Let us now consider the dynamical percolation on this random graph for finite n. To each pair of vertices is attached a Poisson process of intensity $n^{-1/3}$, and every time it rings, one resamples the corresponding edge. Under this process, the collection of connected components undergoes coalescence and fragmentation. Motivated by noise sensitivity questions, we shall study the distributional convergence of this process when n goes to infinity, towards a fragmentation-coalescence process on the continuous limit.

SIZE OF AN EXCURSION SET OF A GAUSSIAN FIELD.

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A smooth Gaussian field $W(\mathbf{p})$, $\mathbf{p} = (x, y)$, having continuous second order partial derivatives, is considered. The locations $\mathbf{p}_i$, $i \geq 1$, of local maxima and minima of the distance $||\mathbf{p}||^2$ from the origin of the points on u-level contours given by the constraint $W(\mathbf{p}) = u$ constitute a point process on the plane. The intensity of the points is given by means of generalized Rice's formula, see [1, 6, 9]. Assume that the points $\mathbf{p}_i$ are ordered according to the distances $||\mathbf{p}_i||$. Let (R, θ) be the polar coordinates of $\mathbf{p}_1$, i.e. the point on contour line $W(\mathbf{p}) = u$ closest to the origin. Then we give a formula for the probability density function for (R, θ). The result is a generalization of the formula for the first passage density given in [7] and can be proved using the generalized Rice's formula, see [5] and [2,3,9].

Additionally, we propose a measure of a size of the excursion set. Let $\mathbf{q}_i$'s be locations of local maxima such that $W(\mathbf{q}_i) > u$, while $\mathbf{p}_i$ be a point on the contour $W(\mathbf{p}) = u$ which is closest to $\mathbf{q}_i$. The Palm distribution of $W(\mathbf{q_i})$ and $||\mathbf{q}_i - \mathbf{p}_i||$ can be used to characterize size of an excursion set. Technical details are given in [5] while Palm distribution of local maximum height can be found in [1].

The results have practical implications for studying the sea elevation that often is modeled by means of a zero mean stationary Gaussian field. A wave crest is located at a positive local maximum of sea elevation, while a steepness is a fraction of the crest height and the horizontal distance of crest location to the zero level contour. The wave steepness and crest height are important variables used to estimate risks for capsizing of vessels, see e.g. [4, 8] for definitions and means to derive distributions of various wave characteristics. The derived formulas will be used to estimate distribution of the so defined wave steepness.

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STOCHASTIC 3D MODELING OF THREE-PHASE MICROSTRUCTURES WITH FULLY CONNECTED PHASES.

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This contribution concerns methods of stochastic geometry applied in modelling of a given physical material, namely it presents a parametric stochastic model for simulation of microstructures consisting of three phases (pores, Nickle and YSZ), where each of the phases is completely connected. In order to obtain complete connectivity in the model, we use connected graphs $G_i = (X_i, E_i)$ as backbones of the three phases, denoted by $\Xi_i,\ i = 1, 2, 3$.

In the basic model, the strong correlation between expected volume fraction of phases Ξ_i and expectation of weighted total edge length of the graphs G_i was used to relate the model parameters quantitatively to volume fractions of the three phases. The model is generalized in two different ways. The first generalization allows to include anisotropy effects in the model. The second one leads to a model which is flexible enough to fit experimental image data with respect to volume fractions, tortuosities and constrictivities. In this generalized model the Nelder-Mead method was used for estimation of all model parameters from experimental image data.

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LOCAL DIGITAL ALGORITHMS FOR ESTIMATION OF INTRINSIC VOLUMES.

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Local algorithms are widely used in the analysis of digital images for estimation of intrinsic volumes. The algorithms are based on counting the number of local configurations of black and white points, which makes them fast and easy to implement. However, with the exception of volume and Euler characteristic estimators, local algorithms are always biased. Recent attempts to define local algorithms for grey-scale images show much better convergence properties. In this talk, we review the known convergence results about local algorithms.

CROSSING PROBABILITIES IN PLANAR VORONOI PERCOLATION.

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Abstract. The planar Voronoi percolation is defined by the following random colouring of the plane. First, consider a Poisson process in the plane with intensity 1 and form its associated Voronoi tiling. Then, colour independenty each tile black with probabilty p, and white otherwise. In this framework, I will present a new Russo-Seymour-Welsh (RSW) result, which provides a bound on the probability to cross rectangles (with a black path) in the long direction, assuming a bound on the probability to cross squares. I will explain how this result offers a clear picture of the phase transition when the density p of black tiles varies. I will also present a joint work with Daniel Ahlberg, Simon Griffith, and Rob Morris, in which we use this new RSW to prove a conjecture of Benjamini, Kalai and Schramm concerning the quenched crossing probabilities in critical Voronoi percolation.