4th Stochastic Geometry Days, August 24 - 28 2015
LMA, Université de Poitiers

PHD students accepted talks, August 26 - 28 2015

QUANTIFYING REPULSIVENESS OF DETERMINANTAL POINT PROCESSES

Christophe Biscio
Laboratoire de Mathématiques Jean Leray, Université de Nantes, France

Advisor: Frédéric Lavancier, Université de Nantes, France

Determinantal point processes (DPPs) have recently proved to be a useful class of models in several areas of statistics, including spatial statistics, statistical learning and telecommunications networks. They are models for repulsive (or regular, or inhibitive) point processes, in the sense that nearby points of the process tend to repel each other. Their growing interest in the statistical community are due to that their moments of any order are explicitly known, parametric families can easily been considered and they can be simulated easily and quickly.

We consider two ways to quantify the repulsiveness of a stationary point process, both based on its second order properties, namely the pair correlation function, and we address the question of how repulsive a stationary DPP can be.

We determine the most repulsive stationary DPP, when the intensity is fixed, and for a given R>0 we investigate repulsiveness in the subclass of R-dependent stationary DPPs, i.e. stationary DPPs with R-compactly supported kernels. Finally, in both the general case and the R-dependent case, we present some new parametric families of stationary DPPs that can cover a large range of DPPs, from the stationary Poisson process (the case of no interaction) to the most repulsive DPP.

CELLS WITH MANY FACETS IN AN HYPERPLANE MOSAIC

Gilles Bonnet
Institute of Mathematics, University of Osnabrück, Germany

Advisor: Matthias Reitzner, Institute of Mathematics, University of Osnabrück, Germany

We consider a stationary Poisson hyperplane process η in $\mathbb{R}^d$ with directional distribution φ. The closure of the connected components of $\mathbb{R}^d \setminus \bigcup_{H\in\eta} H$ are polytopes. The mosaic X_η is the collection of these polytopes, and we call the polytopes the cells of X_η. Almost surely one cell contains the origin in its interior, we call it the zero cell and denote it by Z_0. We investigate the distribution of the random polytope Z_0. In particular we prove (under a weak assumption on φ) that there exist positive constants c_1 and c_2 such that for n big enough we have

$$(c_1 n)^{-2n/(d-1)} < \mathbb{P}(Z_0 \text{ has } n \text{ facets}) < (c_2 n)^{-2n/(d-1)}.$$

If time permits we will show that the same hold if we replace Z_0 by the typical cell Z. It extends similar results of Calka and Hilhorst [1] who gave a more precise asymptotic expansion of the above probability but only in the two dimensional and isotropic case.

Joint work with Pierre Calka and Matthias Reitzner.

References:

[1] H. J. Hilhorst and P. Calka. Random line tessellations of the plane: statistical properties of many-sided cells. *J. Stat. Phys.*, 132(4):627–647, 2008.

FEATURES SELECTION FOR MODELING THE INTENSITY OF SPATIAL POINT PROCESSES

Achmad Choiruddin
Laboratoire Jean Kuntzmann, Université Grenoble Alpes, France

Advisors: Jean-françois Coeurjolly and Frédérique Letué, Université Grenoble Alpes, France

Abstract. Variable selection is fundamentally important for knowledge discovery with high-dimensional data and it could greatly enhance the prediction performance of the fitted model. In addition, variable selection has been a very active research area since Tibshirani introduced least absolute shrinkage and selection operator (LASSO) in penalized likelihood procedure, where this penalization function is added to the likelihood function and used to shrink small coefficient(s) to zero(s) while retaining large coefficient(s) in the model. In this work, we consider the selection of covariates when modeling the intensity of spatial point processes using general penalization function (including Lasso, Group Lasso, Elastic Net, and SCAD Penalty function). In particular, we study the *oracle property* (i.e., performing as well as if the true underlying model were given in advance) of the regularization methods proposed, named regularized weighted estimating equation, which performs simultaneously variable selection and parameter estimation. Such a problem has already been investigated in a recent paper from Thurman, Fu, Guan, and Zhu (Statistica Sinica, 2015), we extend the theoritical part of this study by first considering more general penalty functions and, second, by proposing a more accurate estimator of the asymptotic covariance matrix which takes into account the form of the penalty function. We perform then a simulation study to explore the effectiveness of using the regularization methods in our procedure.

STATIC CLUSTERS IN CELLULAR NETWORKS. THE NEAREST NEIGHBOR MODEL

Luis David Álvarez Corrales
INFRES, Télécom ParisTech, France

Advisors: Anastasios GIOVANIDIS*,Philippe MARTINS** , *CNRS, LTCI, **INFRES, Télécom ParisTech

The problem of base station cooperation in cellular networks has recently been set within the framework of Stochastic Geometry. In this way, the irregularity of network base station locations can be modeled. Existing works consider that a user dynamically chooses the set of stations that cooperate for his/her service. However, this assumption is difficult to be applied in practice. For this reason, it is suggested to group base stations in a static way. The clusters formed do not change over time depending on the needs of different users. Recent literature defines static clusters in a way that is neither systematic nor optimal. A static and performance-optimal clustering procedure should minimise information exchange and infrastructure between base stations.

For the above problem, we propose a grouping method based on node proximity. Specifically, we make use of the Nearest Neighbor Model. To keep things simple, we restrict ourselves to the case where only singles and pairs of base stations are allowed to be formed. For this, two new point process are defined from the dependent thinning of a stationary point process, one for the singles and one for the pairs. For a Poisson point process, structural characteristics for the two processes are provided, such as average proportion, repulsion/atraction, and Palm meaures. An analysis of the Interference fields generated by these two new point processes provides explicit formulas to their expected values and their Laplace transform.

We further analyse an approximation to the first model that mimics the cluster structure of the base stations, using the Poisson point process. The results of the analysis constitute a novel toolbox towards the performance evaluation of networks with static cooperation. It applies to different performance measures related to the so called Signal-to-Interference-and-Noise-Ratio (SINR), such as coverage probability, throughput or delay. A variety of practical cooperation systems, including time-division, MIMO, frequency-reuse, coherent and non-coherent transmission, etc, can be analysed through our approach.

ON THE FINITENESS OF THE SECOND MOMENT OF THE NUMBER OF CROSSINGS BY THE GRADIENT OF A STATIONARY GAUSSIAN RANDOM FIELD

Julie Fournier
MAP5, Université Paris Descartes, France

Advisor: Anne Estrade, Université Paris Descartes, France

Let $X : \Omega \times \mathbb{R}^d \to \mathbb{R}$ be a stationary Gaussian random field. We assume that almost every of its realizations is of class $\mathcal{C}^2$ on $\mathbb{R}^d$. No assumption of isotropy is made. For any bounded rectangle $T \subset \mathbb{R}^d$ and any level $v \in \mathbb{R}^d$, we consider the number $N^{X'}(T,v)$ of points in T where the gradient of X, denoted by X', reaches the value v, i.e. $N^{X'}(T,v) = \#\{t \in T : X'(t) = v\}$. For $v = 0$, it is nothing but the number of stationary points of X in T.

In dimension $d = 1$, the problem of the finiteness of the second moment of the random variable $N^{X'}(T,v)$ has already been investigated and solved. Denoting by r the covariance function of X, it has been proven that $N^{X'}(T,v) \in L^2(\Omega)$ if and only if $t \longmapsto \frac{r^{(4)}(0)-r^{(4)}(t)}{t}$ is integrable in some neighbourhood of 0, condition known as the Geman condition (see [1]).

In dimension $d > 1$, no such condition has ever been found but some results have been proven. For instance, Elizarov gave in [2] a sufficient condition on the covariance function r for the finiteness of the second moment of the number of stationary points. Estrade and León showed in [3] that $\mathrm{Var}\left(N^{X'}(T,v)\right)$ is finite under the additional hypothesis that X is isotropic and of class $\mathcal{C}^3$. My work, in collaboration with Anne Estrade, consists in a generalization of the Geman condition to higher dimensions, under the assumptions made at the beginning of this abstract.

References:

[1] Kratz M., León J.R., *On the second moment of the number of crossings by a stationary Gaussian process.* Ann. Probab., vol 34, 1601-1607 (2006).

[2] Elizarov A.I., *On the Variance of the Number of Stationary Points of a Homogeneous Gaussian Field.* Theory of Probability & Its Applications, Vol. 29, No. 3, 569-570 (1985).

[3] Estrade A., León J.R., *A central limit theorem for the Euler characteristic of a Gaussian excursion set.* Preprint, https://hal.archives-ouvertes.fr/hal-00943054 (2014).

CONTINUUM RANDOM CLUSTER MODEL

Pierre Houdebert
Laboratoire Paul Painlevé, Université Lille 1, France

Advisor: David Dereudre, Université Lille 1, France

The Random Cluster Model(RCM), introduced by Fortuin and Kasteleyn around 1969, is a classical statistical mechanic model where the Hamiltonian is proportional to $q^{N_{cc}(\omega)}$ where q is a positive real number and $N_{cc}(\omega)$ denote the number of connected components of the configuration ω. The RCM appears as the geometrical distribution of several marked models, like the Ising Model, Potts Model or the lattice Widom-Rowlinson Model. In the lattice settings the RCM has been massively studied, see for example [3], and existence and phase transition results has been found. In the continuum settings, finite volume Continuum Random Cluster Model(C-RCM) has been used to give new phase transition proofs for the Continuum Potts Model, see [1] and [2]. But no existence or phase transition results were found for the infinite volume C-RCM.

In my talk I will first start by introducing the C-RCM in the finite volume case and then in the infinite volume case through the DLR equation. I will then give the first existence result which states that for integrable radii law, we have the existence of a C-RCM, and I will give a sketch of the proof. This proof does not extend in the case of non integrable radii but by using the Widom-Rowlinson Model and its relation with the C-RCM, we get an existence and phase transition result in the special assumption of non integrable radii law. This is a joint work with David DEREUDRE, Lille 1 University.

References:

[1] J. T. Chayes, L. Chayes, and R. Kotecký. The analysis of the widom-rowlinson model by stochastic geometric methods. *Comm. Math. Phys.*, 172(3):551–569, 1995.

[2] H.-O. Georgii and O. Häggström. Phase transition in continuum Potts models. *Comm. Math. Phys.*, 181(2):507–528, 1996.

[3] Geoffrey Grimmett. *The random-cluster model*, volume 333 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 2006.

ASYMPTOTICS OF SUPERPOSITION OF POINT PROCESSES

Aurélien Vasseur
INFRES, Télécom ParisTech, France

Advisor: Laurent Decreusefond, Télécom ParisTech, France

The characteristic independence property of Poisson point processes gives an intuitive way to explain why a sequence of point processes becoming less and less repulsive can converge to a Poisson point process. Our aim is to show this convergence for sequences built by superposing, thinning or rescaling determinantal processes. We use Papangelou intensities and Stein's method to prove this result with a topology based on total variation distance.