Lectures on Stochastic Methods for Image Analysis

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LECTURE 1

VISUAL PERCEPTION:
GESTALT THEORY AND THE NON ACCIDENTALNESS PRINCIPLE
PART 0 : INTRODUCTION
Introduction
Helmholtz Principle, also called the **non-accidentalness principle**, can be stated in two different ways:

1. The first way is common sensical. It simply states that "we do not perceive any structure in a uniform random image". (In this form, the principle was first stated by Attneave in 1954).

2. In its stronger form, the Helmholtz principle states that whenever some large deviation from randomness occurs, a structure is perceived. In other words: "we immediately perceive whatever has a low likelihood of resulting from accidental arrangement". (Stated in Computer Vision by S.-C. Zhu or D. Lowe)
What’s make an image an image and not noise?
What structures are we looking for?

Not all possible structures are relevant for visual perception.
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- Not all possible structures are relevant for visual perception.
- They have to be fixed before - and not after the observation.
- The relevant structures for visual perception have been studied by the Gestalt School of Psychophysiology.
Visual Perception

How do we perceive geometric objects in images?

What are the laws and principles of visual construction?
In other words, how do you go from pixels (or retina cells) to visual objects (lines, triangles, etc.)?
PART I : GESTALT THEORY OF VISUAL PERCEPTION
Before Gestalt Theory: optic-geometric illusions

The aim of these illusions is to ask: “what is the reliability of our visual perception?”

A first example: Zoellner’s Illusion (1860)
Other examples of optic-geometric illusions

Hering’s Illusion (1861)

Are lines a and b straight?
Other examples of optic-geometric illusions

Müller-Lyer’s Illusion (1889)
Other examples of optic-geometric illusions

Sander’s Illusion

(a)

(b)

(c)
Gestalt Theory

- Gestalt theory does not continue on the same line. The question is not why we sometimes see a distorted line when it is straight; the question is why we do see a line at all. This perceived line is the result of a construction process whose laws it is the aim of Gestalt theory to establish.

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- Gestalt theory (Wertheimer, Metzger, Kanizsa) starts with the assumption of (a small list of) active grouping laws in visual perception: vicinity, same attribute (like colour, shape, size or orientation), alignment, good continuation, symmetry, parallelism, convexity, closure, constant width, amodal completion, T-junctions, X-junctions, Y-junctions.

- M. Wertheimer, _Unterzuchungen zur lehre der gestalt_, Psychologishe Forshung (1923).
- W. Metzger, _Gesetze des Sehens_, Kramer, 1953.
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- The above listed grouping laws belong, according to Kanizsa, to the so-called primary process, opposed to a more cognitive secondary process.

- M. Wertheimer *Unterzuchungen zur lehre der gestalt*, Psychologishe Forschung (1923).
Seeing and thinking

Figure: Grammatica del Vedere, Gaetano Kanizsa.
Seeing and thinking

Figure: Grammatica del Vedere, Gaetano Kanizsa.
Elementary grouping laws

Vicinity
Elementary grouping laws

Same color/size/orientation
Elementary grouping laws

Closure
Elementary grouping laws

Symmetry
Elementary grouping laws

Good continuation
T- and X- junctions

T-junctions
T- and X- junctions

T-junctions
T- and X- junctions

T-junctions
T- and X- junctions

X-junctions
Amodal Completion
Amodal Completion
Amodal Completion
Impossible Figures

Visual grouping laws are stronger than physical sense. Perspective effect is created by **Y-junctions**.

**Penrose Triangle**
Impossible Figures

Visual grouping laws are stronger than physical sense. Perspective effect is created by **Y-junctions**.

**Penrose Fork**
... and past experience?
... and past experience?
... and past experience?
Recursivity

All grouping Gestalt laws are *recursive*: they can be applied first to atomic inputs and then in the same way to partial Gestalts already constituted.

The same partial Gestalt laws namely alignment, parallelism, constant width and proximity, are recursively applied not less than six times.
Conflicts between grouping laws
Conflicts between grouping laws

Chiusura contro buona continuazione.
Conflicts between grouping laws

1.55 La continuità di direzione è più forte della chiusura.
Conflicts between grouping laws

Se la tendenza alla massima simmetria avesse la stessa importanza nel vedere che ha nel pensare, questa configurazione sarebbe vista come la giustapposizione delle due forme simmetriche di figura 2.30.
Conflicts between grouping laws
Masking **by texture**

“The whole masks its parts”.
Masking phenomenon

Masking by addition
Masking phenomenon

Masking by subtraction

A masking is efficient if a new grouping law appears.
Masking phenomenon

Masking by articulation shape/background

Kanisza: “il fondo non é forma” (« The background has no shape »).
PART II : THE NON-ACCIDENTALNESS PRINCIPLE
Helmholtz Principle (or Non-accidentalness Principle)

Helmholtz Principle :

- « We don’t perceive any structure in a noise image » (Attneave).
- « We immediately perceive any structure that is too regular to be there just by chance » (Zhu, Lowe)

Examples :

- parallel straight lines in 2D are perceived as the projection of parallel 3D lines.
- if you play dice and you obtain a sequence 6,6,6,6,6 - you will certainly notice it !

→ The principle is used to compute *detectability thresholds*. 
Example: Birthdays in a class

**Question**: In a class of 30 students, is it surprising if $n$ of them have the same birthday?

**Hypothesis**: birthdays = random variables, independent and identically uniformly distributed on the 365 days of the year.

Let $C_n$ be the number of groups of $n$ students having the same birthday. Let $P_n = P[C_n \geq 1]$ and $p_n = P[C_n \geq 1 \text{ and } C_{n+1} = 0]$.

We have $E[C_n] = \text{expected value of the number of groups of } n \text{ students having the same birthday}$.

$\rightarrow$ What are the values of $P_2, E[C_2]$? And more generally of $P_n, E[C_n]$?
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\( \rightarrow \) What are the values of \( P_2, E[C_2] \)? And more generally of \( P_n, E[C_n] \)?

One can compute: \( P_2 = 1 - \frac{365 \times 364 \times \ldots \times 336}{365^{30}} \approx 0.706. \)
\( \rightarrow \) it’s not a big surprise to have two students with the same birthday!
Birthdays in a class

We can also compute:

\[ P_3 = P_2 - p_2 = P_2 - \frac{1}{365^{30}} \sum_{i=1}^{15} \left[ \frac{\prod_{j=0}^{i-1} \left( \frac{30-2j}{i!} \right)}{\prod_{k=0}^{29-i} (365 - k)} \right] \approx 0.028. \]

→ As \( n \) grows, \( P_n \) is more and more uneasy to compute.
Birthdays in a class

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→ As \( n \) grows, \( \mathbb{P}_n \) is more and more uneasy to compute.

But the expectation is much simpler:

\[
\mathbb{E}[C_n] = \mathbb{E} \left[ \sum_{1 \leq i_1 < \cdots < i_n \leq 30} \mathbb{1}_{\{i_1, \ldots, i_n \ \text{have the same birthday}\}} \right]
\]

\[
= \sum_{1 \leq i_1 < \cdots < i_n \leq 30} \mathbb{P}[i_1, \ldots, i_n \ \text{have the same birthday}] = \frac{1}{365^{n-1}} \binom{30}{n}.
\]
Birthdays in a class

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→ As $n$ grows, $P_n$ is more and more uneasy to compute.

But the **expectation** is much simpler:

$$\mathbb{E}[C_n] = \mathbb{E} \left[ \sum_{1 \leq i_1 < \cdots < i_n \leq 30} 1 \{ i_1, \ldots, i_n \text{ have the same birthday} \} \right]$$

$$= \sum_{1 \leq i_1 < \cdots < i_n \leq 30} P[i_1, \ldots, i_n \text{ have the same birthday}] = \frac{1}{365^{n-1}} \binom{30}{n}.$$ 

Thanks to **Markov Inequality**, we always have $P_n \leq \mathbb{E}[C_n]$.

Numerical computations give:

- $P_2 = 0.706$, and $\mathbb{E}[C_2] = 1.192$;
- $P_3 = 0.0285$, and $\mathbb{E}[C_3] = 0.0347$;
- $P_4 = 5.3 \times 10^{-4}$, and $\mathbb{E}[C_4] = 5.6 \times 10^{-4}$.
PART III : COMBINING GESTALT THEORY AND THE NON-ACCIDENTALNESS PRINCIPLE
General formulation of \textit{a contrario} methods

\textbf{Given} : \( n \) geometric objects \( O_1, \ldots, O_n \). Let \( X_i \) be a random variable describing an attribute of the \( O_i \) (for instance : position, color, orientation, size, etc...).

\textbf{Hypothesis} \( \mathcal{H}_0 \) (also called \textit{background distribution} or \textit{noise model} or \textit{a contrario model}) : \( X_1, \ldots, X_n \) are independent identically distributed.

\textbf{Observed event} \( E \) on \( X_1, \ldots, X_k \) (ex : \( X_1, \ldots, X_k \) are similar).

Can this observed event happen by chance ? (= how likely is it under the null hypothesis \( \mathcal{H}_0 \) ?)

\textbf{Test} :

\[
\text{NFA}(E) := \mathbb{E}_{\mathcal{H}_0}[\text{nb of occurrences of } E] \leq \varepsilon
\]

If the test is positive, then the observed event \( E \) is said to be an \( \varepsilon \)-meaningful event.

Example: A black square in a Bernoulli noise

Hypothesis $\mathcal{H}_0$: binary image of size $L \times L$, pixels i.i.d. $\sim$ Bernoulli of parameter $p$.

Observation: we observe a black square of size $l_0 \times l_0$ pixels somewhere in the image.

$\mathbb{P}[\text{a given square of side length } l \text{ is all black}] = p^l$.

$\mathbb{E}[\text{Nb of black squares with side length } l \text{ in the image}] = p^l (L - l)^2$.

$\mathbb{E}[\text{Nb of black squares with side length } l \geq l_0] = \sum_{l \geq l_0} p^l (L - l)^2$.

Questions:

- How to fix the value of $p$?
- Why considering only squares?
Example: A black square in a Bernoulli noise

$L = 256, p = 0.5$
$l_0 = 3$. NFA = 125.9
Example: A black square in a Bernoulli noise

\[ L = 256, \ p = 0.5 \]
\[ l_0 = 5, \ \text{NFA} = 0.001 \]
Exercise 1: Gestalt comment of this figure
Exercise 2 : Gradient orientation in noise image

The gradient in a (discrete) image $u$ is computed at a pixel of coordinates $(x, y)$ by

$$\nabla u(x, y) = \frac{1}{2} \left( \begin{array}{c} u(x + 1, y + 1) + u(x + 1, y) - u(x, y + 1) - u(x, y) \\ u(x + 1, y + 1) + u(x, y + 1) - u(x + 1, y) - u(x, y) \end{array} \right)$$

$$= \|\nabla u(x, y)\| e^{i\theta_u(x, y)}$$

1) What is the law of $\theta_u(x, y)$ when $u$ is a Bernoulli noise image?

2) What is the law of $\theta_u(x, y)$ when $u$ is a white noise image (meaning that the grey levels are i.i.d. $\mathcal{N}(0, \sigma^2)$)?