Static clusters in cellular Networks. The Nearest Neighbor Model.

Luis David Álvarez Corrales
with **Anastasios Giovanidis** & **Laurent Decreusefond.**

Télécom ParisTech, France.

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Introduction

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- In recent years, it became popular to model the irregularity of the network base station (BS) locations within the framework of Stochastic Geometry.
- Cooperation in Cellular Networks: Very important subject of study. Why? Two or more BSs exchange user information to offer stronger signal with reduced interference.
    - Coverage improvement.
    - Better service to cell-edge users (inter-cell interference).

Introduction

Benefit depends on:

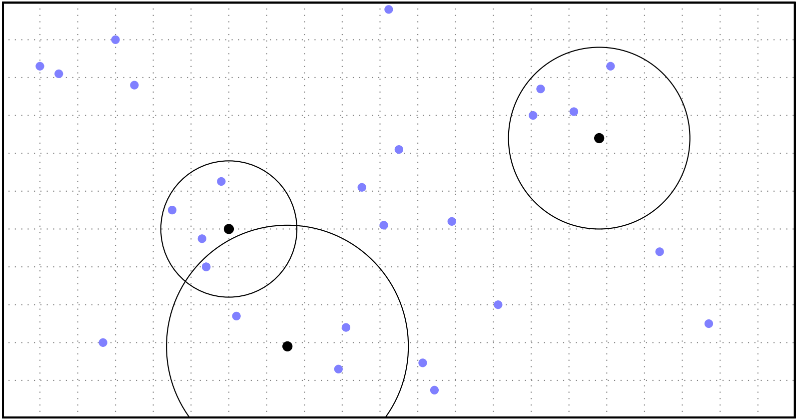
- ☞ Cooperation type.
- ☞ Information exchange.
- ✍ **Number and position of cooperating nodes!**

Literature

Stochastic Geometry (PPP) models:

- Andrews, Baccelli & Ganti (TCOM 2011): Tractable Approach
- Dhillon, Ganti, Baccelli & Andrews (JSAC 2012): K-tier HetNets
- Akoum & Heath (TSP 2013): Random layered clustering and cooperation.
- Baccelli & Giovanidis (Asilomar 2013 - TWC 2015): Cooperation between base station pairs.
- Nigam, Minero & Haenggi (TCOM 2014): Cooperation in groups of more than two.
- Tanbourgi, Singh, Andrews & Jondral (TWC 2014): Circle of fixed radius around user.
- Keeler, Błaszczyszyn & Karray (ISIT 2013, arXiv 2014): k-coverage, factorial moment measures.

Dynamic clusters: User-Driven



Every user (black dots) is served by its four closest BSs (blue dots).

Static Vs Dynamic

Problems with Dynamic Clusters:

- Each user can ask any set of BSs to cooperate → overburdens the backhaul/control channel with intensive communication.
- Time- (or more general Resource-) sharing between clusters.

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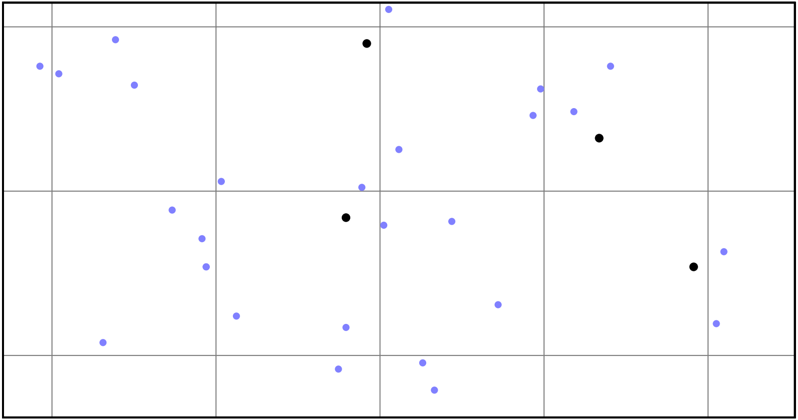
Could **static** cooperation groups be more realistic? YES!

- Reasonable information exchange between BSs.
- No resource-sharing.
- One controller per cluster: Better to program and coordinate.

Static Clusters: Randomized & Hierarchical



Clusters are formed by those BSs (blue dots) inside each square.

Static Clusters: Proximity

☞ Define a-priori static groups by means of proximity.

✍ Why?

- Strong signal.
- Weak interference (avoid first-order).
- Fast coordination.
- BSs share a planar area of common interest.

Here: Singles and Pairs

The maximum group size constraint is $K = 2$:

- **single** BSs operating individually,
- **pairs** of BSs providing service cooperatively.

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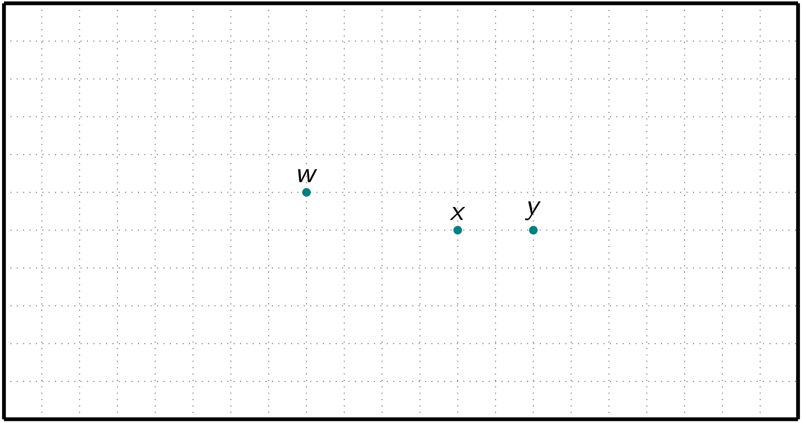
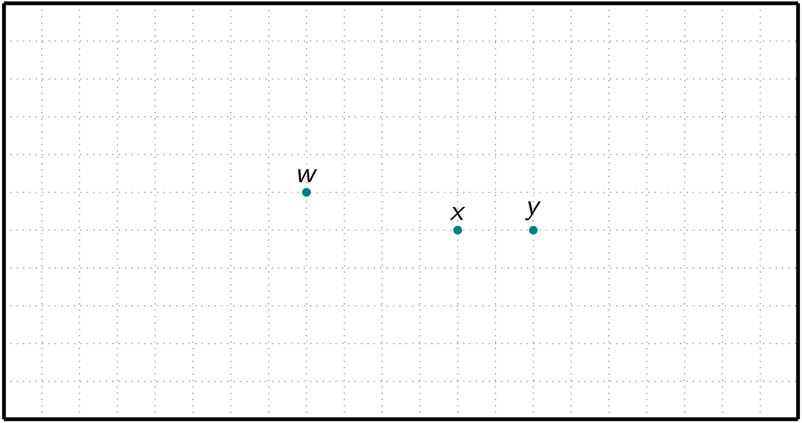
Why?

- To be avoided: big number of BSs serving only one user.
- Easier to derive first results.

Mutually Nearest Neighbour Pair



Single



The point processes $\Phi^{(1)}$ and $\Phi^{(2)}$

Definition

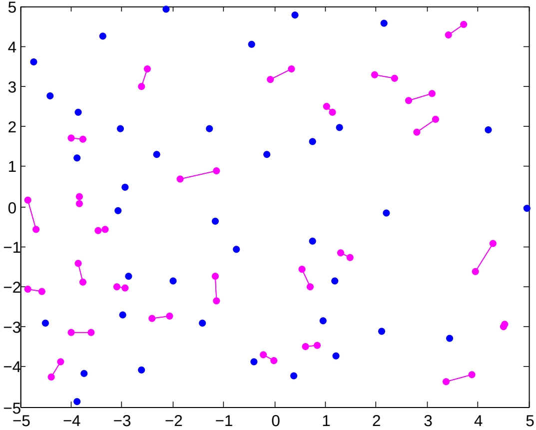
Fixed a realisation ϕ,

$$\phi^{(1)} = \{x \in \phi \;\;\&\;\; x \text{ is single } \},$$
$$\phi^{(2)} = \{x \in \phi \;\;\&\;\; x \text{ cooperates with another element of } \phi\}.$$

Given a stationary point process $\Phi = \{\phi\}$ we define the point process $\Phi^{(1)}$ and $\Phi^{(2)}$ by means of the dependent thinning defined above:

$$\Phi^{(1)} = \{\phi^{(1)}\}$$
$$\Phi^{(2)} = \{\phi^{(2)}\}.$$

The point processes $\Phi^{(1)}$ and $\Phi^{(2)}$



Some results.

If Φ is an homogeneus PPP,

- 62% of points are pairs, and 38% are single points (it is impossible for $\Phi^{(1)}$ and $\Phi^{(2)}$ to be PPP).
- A representations for the Palm measures of $\Phi^{(1)}$ and $\Phi^{(2)}$.
- A closed formula for the NN function of $\Phi^{(2)}$.

Some numerical results:

- Repulsion between the process of singles, and attraction between the process of pairs.
- Tight bounds for the ES funcions of $\Phi^{(1)}$ and $\Phi^{(2)}$.
- Average Voronoi surface proportions.

Interference field

Interference field generated by $\Phi^{(1)}$ and $\Phi^{(2)}$:

$$
\begin{aligned}
\mathcal{I}^{(1)} &= \sum_{x \in \Phi^{(1)}} f(x), \\
\mathcal{I}^{(2)} &= \sum_{x \in \Phi^{(2)}} \sum_{y \in \Phi^{(2)}} \frac{1}{2} g(x,y) 1_{\left\{x \overset{\Phi}{\leftrightarrow} y\right\}}.
\end{aligned}
$$

Expected Interference

Theorem

The expected value of the interference fields has the analytical representation

$$\mathbb{E}\left[\mathcal{I}^{(1)}\right] = \int_{\mathbb{R}^2} \mathbb{E}[f(x)](1 - p^*)\lambda dx,$$
$$\mathbb{E}\left[\mathcal{I}^{(2)}\right] = \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \frac{1}{2}\mathbb{E}[g(x,y)]e^{-\lambda|x-y|^2(2\pi-\gamma)}\lambda dy\lambda dx.$$

Laplace Transform for Singles

Theorem

For a random field $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^+$, and $A \subset \mathbb{R}^2$. Let $F^{(n)}(x_1, \ldots, x_n) = (f(x_1), \ldots, f(x_n))$, the LT of the interference $\mathcal{I}^{(1)}$ generated by f and $\Phi_A^{(1)}$ is equal to

$$\mathbb{E}\left[e^{-s\mathcal{I}^{(1)}}\right] = e^{-\lambda \mathcal{S}(A)}\Bigg(1 + \lambda \int_A \mathbb{E}\left[e^{-sf(x)}\right] dx + \frac{\lambda^2}{2} + \sum_{n=3}^{\infty} \frac{\lambda^n}{n!} \int_A \ldots \int_A \mathbb{E}\left[e^{-s(F^{(n)} \cdot H^{(n)})(x_1, \ldots, x_n)}\right] dx_1 \ldots dx_n\Bigg).$$

Laplace Transform for Doubles

Theorem

For a random field $g : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}^+$, and $A \subset \mathbb{R}^2$. Let $G_i^{(n)}(x_1, \ldots, x_n) = (g(x_i, x_1), \ldots, g(x_i, x_n))$, and $G^{(n)} = (G_1^{(n)}, \ldots, G_n^{(n)})$, the LT of the interference $\mathcal{I}^{(2)}$ generated by g and $\Phi_A^{(2)}$ is equal to

$$\mathbb{E}\left[e^{-s\mathcal{I}^{(2)}}\right] = e^{-\lambda\mathcal{S}(A)}\Bigg(1 + \lambda\mathcal{S}(A) + \frac{\lambda^2}{2}\int_A\int_A \mathbb{E}\left[e^{-\frac{s}{2}(g(x,y)+g(y,x))}\right]\lambda dy \lambda dx + \sum_{n=3}^{\infty}\frac{\lambda^n}{n!}\int_A \ldots \int_A \mathbb{E}\left[e^{-\frac{s}{2}(G^{(n)}\cdot J^{(n)})(x_1,\ldots,x_n)}\right] dx_1 \ldots dx_n\Bigg).$$

Extensions.

- Higher moments of $\Phi^{(1)}$ and $\Phi^{(2)}$.
- A rigorous proof for the numerically results.
- Use the above results to analyze the performance of cooperating cellular networks.
- Cooperation functions.

Extesions

$(v_x)_{x \in \mathbb{R}^2}$ random propagation effects from the BS x to the typical user.

- No cooperation function, $f(x) = \frac{v_x}{||x||^\beta}$.
- Pair cooperation function,

$$g(x,y) = \begin{cases} \frac{v_x}{||x||^\beta} + \frac{v_y}{||y||^\beta}, & \text{[NC]} \\ \max\left\{\frac{v_x}{||x||^\beta}, \frac{v_y}{||y||^\beta}\right\}, & \text{[OF1]} \\ \mathbb{1}_{on_x}\frac{v_x}{||x||^\beta} + (1 - \mathbb{1}_{on_x})\frac{v_y}{||y||^\beta}, & \text{[OF2]} \\ \left|\sqrt{\frac{v_x}{||x||^\beta}}e^{i\theta(x)} + \sqrt{\frac{v_y}{||y||^\beta}}e^{i\theta(y)}\right|^2 & \text{[PH].} \end{cases}$$

What kind of laws are we interested in?

Extensions

- Because of the difficulties in analysing the previous schemes, we have further proposed a model that imitates the cluster structure of the BSs:
  - Based on using a superposition of 2 Poisson processes.
  - One is for the singles.
  - The second for the pairs.
- The percentage of single and pairs.
- The distance between two cooperating BSs.

We have obtained closed, analytic formulas.

Thank you for your attention.

☞ luis.alvarez-corrales@telecom-paristech.fr