Mathematisches Forschungsinstitut Oberwolfach

Report No. 51/2015

DOI: 10.4171/OWR/2015/51

# Mini-Workshop: Singular Curves on K3 Surfaces and Hyperkähler Manifolds

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8 November – 14 November 2015

Abstract.

## Introduction by the Organisers

*Acknowledgement:*

**Workshop: Mini-Workshop: Singular Curves on K3 Surfaces and Hyperkähler Manifolds**

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## Abstracts

### Recent progress on the classification of automorphisms of hyperkähler manifolds

Samuel Boissière

The group $\mathrm{Aut}(X)$ of biholomophisms of an irreducible holomorphic symplectic manifold $X$ is a discrete complex Lie group. In this talk I focus on finite subgroups of $\mathrm{Aut}(X)$ where $X$ is a K3 surface or a deformation of the Hilbert scheme $\mathrm{K3}^{[n]}$ of $n$ points on a K3 surface.

The symplectic automorphisms are those who act trivially on the symplectic two-form. All finite groups of symplectic automorphims of K3 surfaces have been classified by Nikulin [12] and Mukai [11]. A generalisation of this classification for deformations of $\mathrm{K3}^{[2]}$ has been obtained by Mongardi [10] and Höhn–Mason [9]. Of special interest is the result of Mongardi, answering a conjecture of Camere [7], showing that every symplectic involution can be deformed to an involution of a Hilbert scheme of two points induced by a symplectic involution of the underlying K3 surface.

Let now $G$ be any finite subgroup of $\mathrm{Aut}(X)$. Looking at the action on the symplectic form, one gets an exact sequence

$$0 \longrightarrow G_0 \longrightarrow G \longrightarrow \mathbb{Z}/m\mathbb{Z} \longrightarrow 0$$

where $G_0$ contains only symplectic automorphisms and $m$ is the non-symplectic index of $G$. If $X$ is non-projective then $m = 1$ (see [2]). Otherwise, a bound for $m$ is given by $\varphi(m) \leq b_2(X) - \rho(X)$, where $\varphi$ is the Euler's totient function, $b_2(X)$ is the second Betti number of $X$ and $\rho(X) \geq 1$ is its Picard number. The situation is particularly interesting when $m = p$ is a prime number: if $X$ is a K3 surface, then $p \leq 19$ ; but if $X$ is a deformation of $\mathrm{K3}^{[n]}$ one has $p \leq 23$. Boissière–Camere–Mongardi–Sarti [4] have shown the existence of a unique variety in this deformation class that contains a non-symplectic automorphism of order 23. Non-symplectic involutions have been studied and classified by Beauville [3] and Ohashi–Wandel [13].

I will present the main ingredients of the classification, obtained by Boissière–Camere–Sarti [5], of non-symplectic groups $G$ of automorphisms of prime order $p$ such that $3 \leq p \leq 19$, acting on a deformation of $\mathrm{K3}^{[2]}$. The classification is governed by two primitive sublattices of the second cohomology lattice $H^2(X, \mathbb{Z})$, equipped with the Beauville-Bogomolov–Fujiki quadratic form: the sublattice T(G) invariant by the automorphism, and its orthogonal complement S(G). In order to get a classification of the pair of lattices $(\mathrm{T(G)}, \mathrm{S(G)})$, we use on one side deep lattice theorical results on existence of lattices with given signature and discriminant and of embedding of lattices with given orthogonal complement, and on the other side topological information on the fixed locus $X^G$ which is closely related to some numerical invariants of the lattices T(G) and S(G):

the Euler characteristics of $X^G$ is computed by applying the Lefschetz topological fixed point formula, and the sum of the dimensions of the mod $p$ cohomology of $X^G$ is computed by using a formula of Boissière–Nieper-Wißkirchen–Sarti [6] originated in Smith theory and making use of the classification theorem of finite-dimensional $\mathbb{F}_p[G]$-modules of Diederichsen and Reiner. This classification reveals two interesting features:

- in contrary to the classification of non-symplectic automorphisms on K3 surfaces by Nikulin [12] and Artebani–Sarti–Taki [1], in our situation the pair $(\mathrm{T(G)}, \mathrm{S(G)})$ does not determine uniquely the geometry of the fixed locus;
- in contrary to the situation for symplectic involutions on deformations of $\mathrm{K3}^{[2]}$ recalled above, it is not true that non-symplectic automorphisms of prime order $p \geq 3$ are deformations of natural automorphisms on $\mathrm{K3}^{[2]}$.

To illustrate these features I will give some geometric examples using Hilbert schemes of points on K3 surfaces and Fano varieties of lines on cubic fourfolds, endowed with a non-symplectic automorphism of order three.

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